Using controlling chaos technique to suppress self-modulation in a delayed feedback traveling wave tube oscillator

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Controlling chaos

An idea of *controlling chaos* technique for stabilizing of unstable periodic orbits in dynamical systems was suggested by Ott, Grebogy and Yorke (PRL **64**, No. 11. P. 1196-1199 (1990))

A simple and effective method of chaos control by time-delayed feedback named *time-delay autosynchronization* (TDAS) was introduced by K. Pyragas (Phys. Lett. A **170**, No. 6. P. 421-428 (1992))

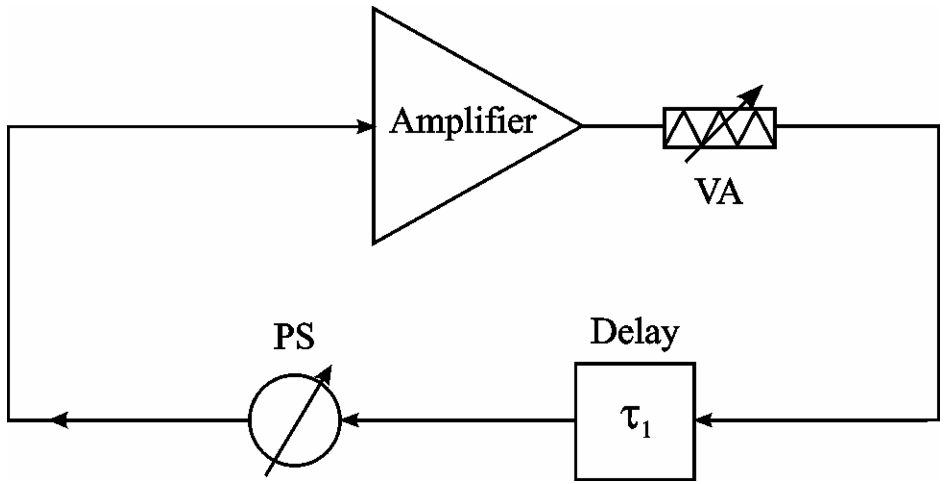
$$\frac{d\mathbf{x}}{dt} = \mathbf{F}\left[\mathbf{x}(t)\right]$$ — dynamical system with chaotic dynamics

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}\left[\mathbf{x}(t)\right] + \varepsilon\left(\mathbf{x}(t) - \mathbf{x}(t-T)\right)$$ — system with time-delayed control with delay time equal to period of motion to be stabilized

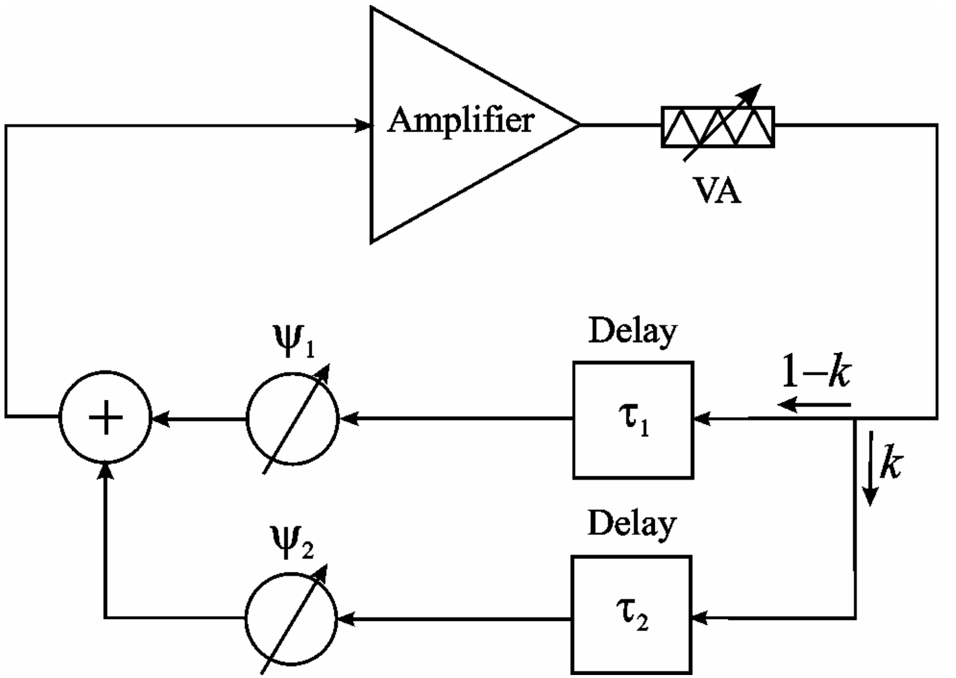
Usually T is unknown a priori
Does not allow to stabilize high-frequency motion

A.M. Dolov and S.P. Kuznetsov (Tech. Phys. **73**, No. 8. P. 139-142 (2003)) — suppress self-modulation in a microwave vacuum tube oscillator via modulation of electron beam current by external feedback control signal with delay time which depends on self-modulation period.

Delayed feedback oscillator



Method of chaos control



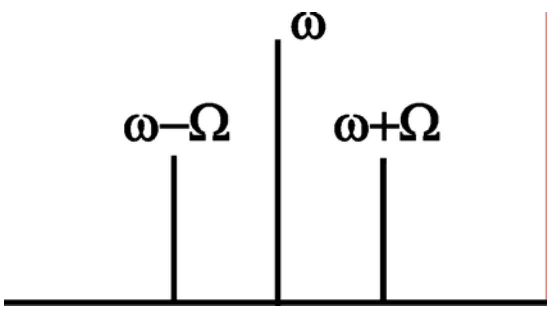
$$A_{in}(t) = \rho(1-k)\,A_{out}(t-\tau_1)\,e^{i\psi_1} + \rho k A_{out}(t-\tau_2)\,e^{i\psi_2}$$

k — control parameter, ρ — attenuation produced by the VA

Idea: To choose delay times and phases so that fundamental waves passing through two feedback legs appear in same phase, while the self-modulational sidebands appear in anti-phase and suppress each other.

Method of chaos control

Consider propagation of a modulated signal



$$A(t)=\left[A^{(\omega)}+A^{(\omega+\Omega)}e^{i\Omega t}+A^{(\omega-\Omega)}e^{-i\Omega t}\right]e^{i\omega t}$$

Substituting into the boundary condition one can show that if we adjust the parameters as

$$\psi_2-\psi_1-\omega\left(\tau_2-\tau_1\right)=2\pi n$$

$$\Omega\left(\tau_1-\tau_2\right)=2\pi m+\pi$$

we obtain

$$A_{in}^{(\omega)}=\rho\left[1-k+k\right]e^{i\left(\psi_1-\omega\delta_1\right)}A_{out}^{(\omega)}=\rho e^{i\left(\psi_1-\omega\delta_1\right)}A_{out}^{(\omega)}$$

— same as for the oscillator with single feedback. *Non-invasive* control.

and

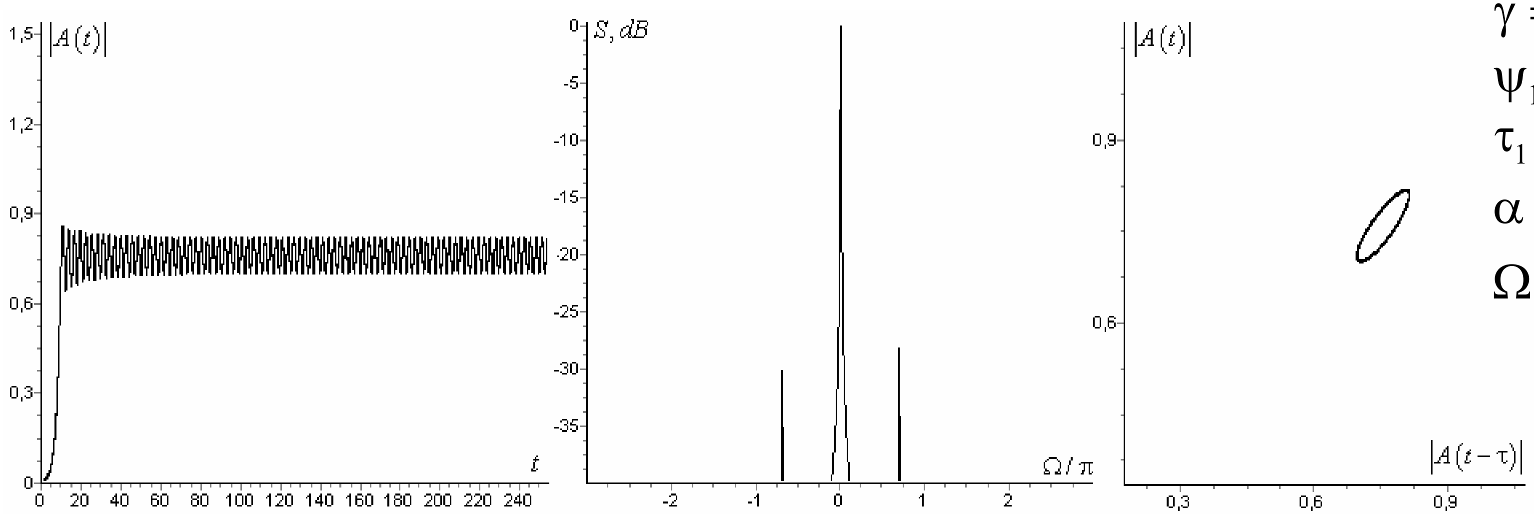
$$A_{in}^{(\omega\pm\Omega)}=\rho\left(1-2k\right)e^{i\left(\psi_1-(\omega\pm\Omega)\delta_1\right)}A_{out}^{(\omega\pm\Omega)}$$

Sideband waves coming from different feedback legs weaken and for k=1/2 completely suppress each other

Delayed feedback oscillator with cubic nonlinearity

$$\frac{dA}{dt}+\gamma A=\alpha\left[(1-k)\left(1-\left|A(t-\tau_1)\right|^2\right)A(t-\tau_1)\,e^{i\psi_1}+k\left(1-\left|A(t-\tau_2)\right|^2 A(t-\tau_2)\,e^{i\psi_2}\right)\right]$$

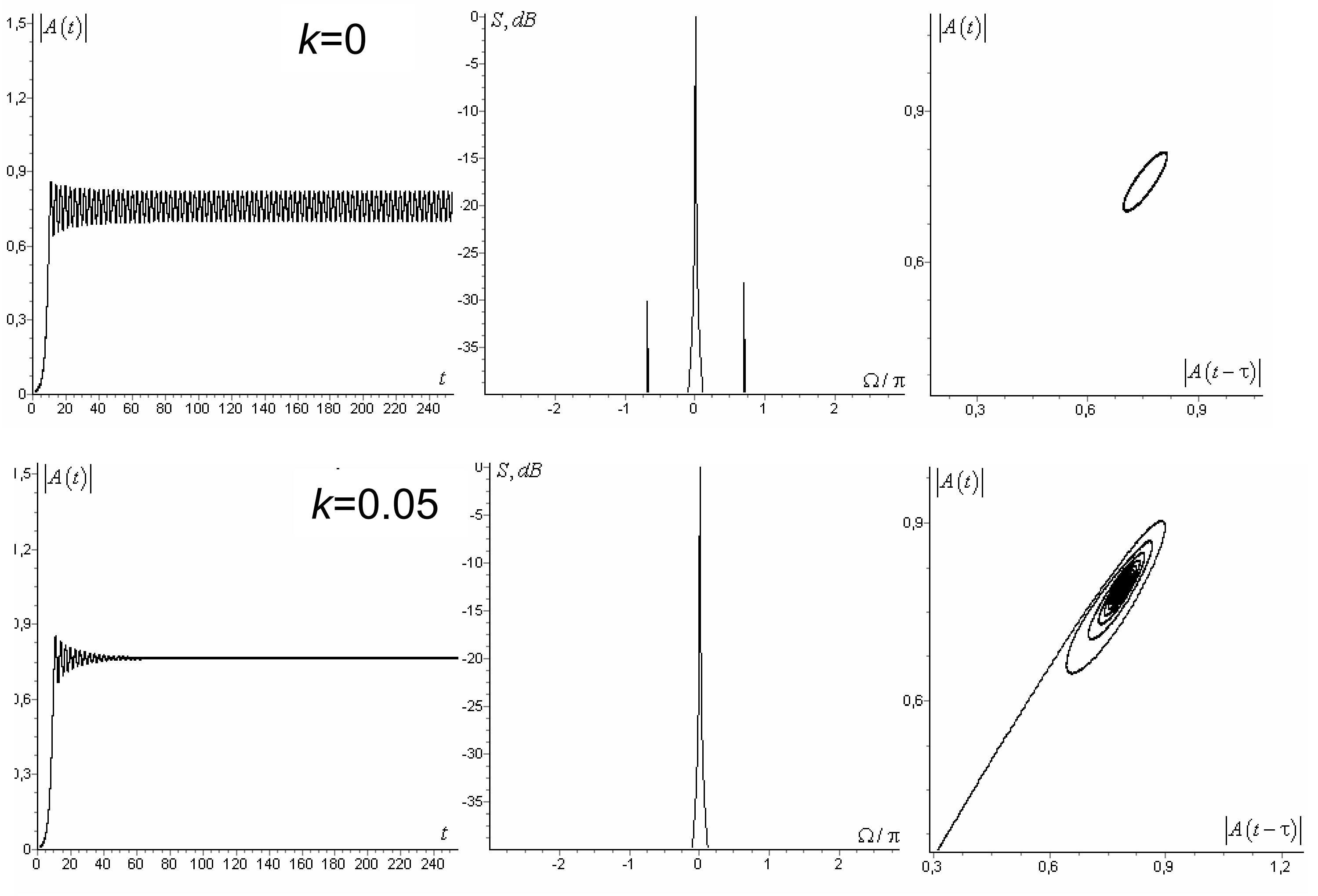
Nonlinear dynamics of a single-feedback oscillator (k=0) was studied in details in N.M. Ryskin, A.M. Shigaev. Complex Dynamics of a Simple Distributed Self–Oscillatory Model System with Delay, Technical Physics **47**, 795-802 (2002).



$$\gamma = 1.5$$
$$\psi_1 = 0.01\pi$$
$$\tau_1 = 1$$
$$\alpha = 3.6$$
$$\Omega \approx 0.7\pi$$

With the increase of α — self-excitation → single-frequency generation → self-modulation → period doublings → chaos

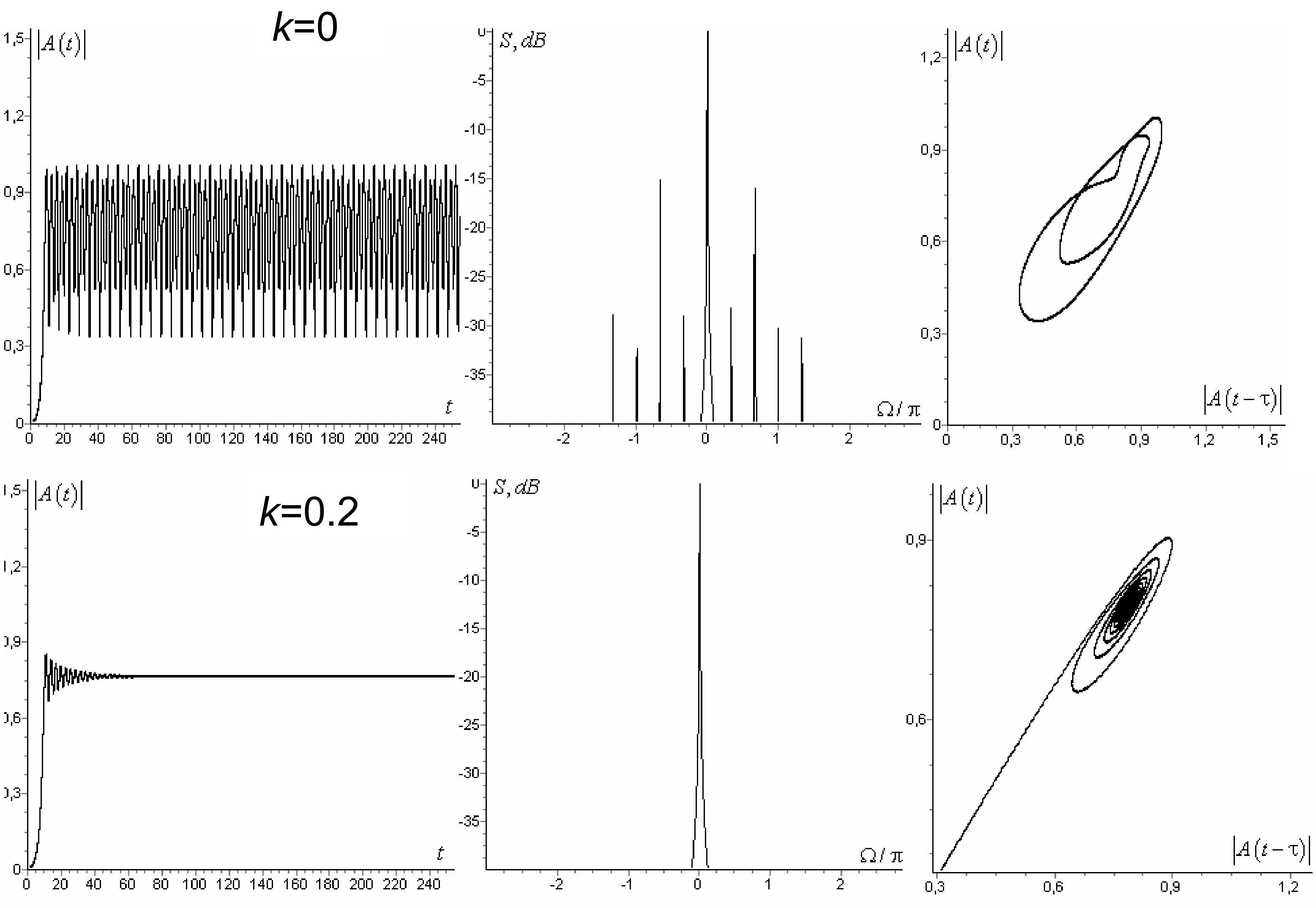
Suppressing of self-modulaton



According to derivations presented above we must choose τ_2=2.43, ψ_2=2.02π

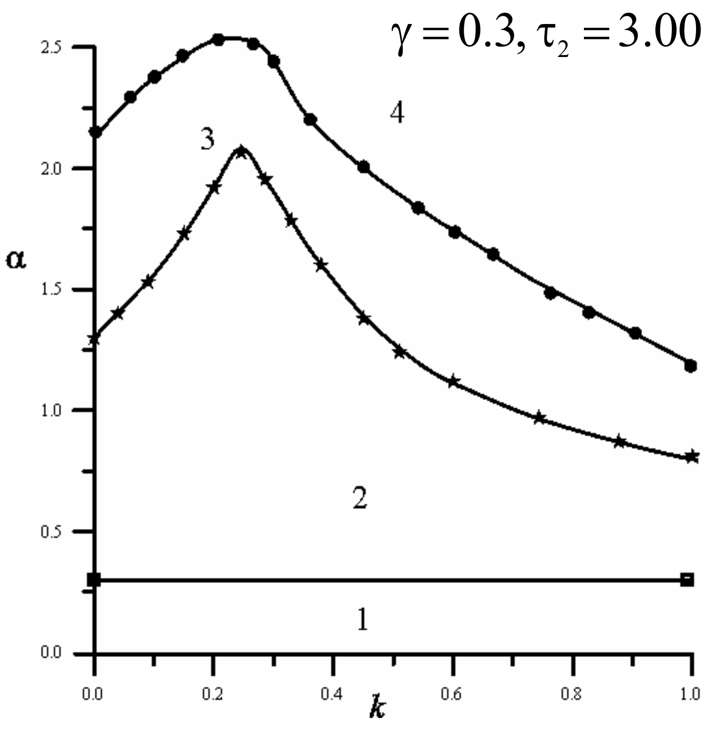
Adding of the secondary control feedback allows to suppress self-modulation

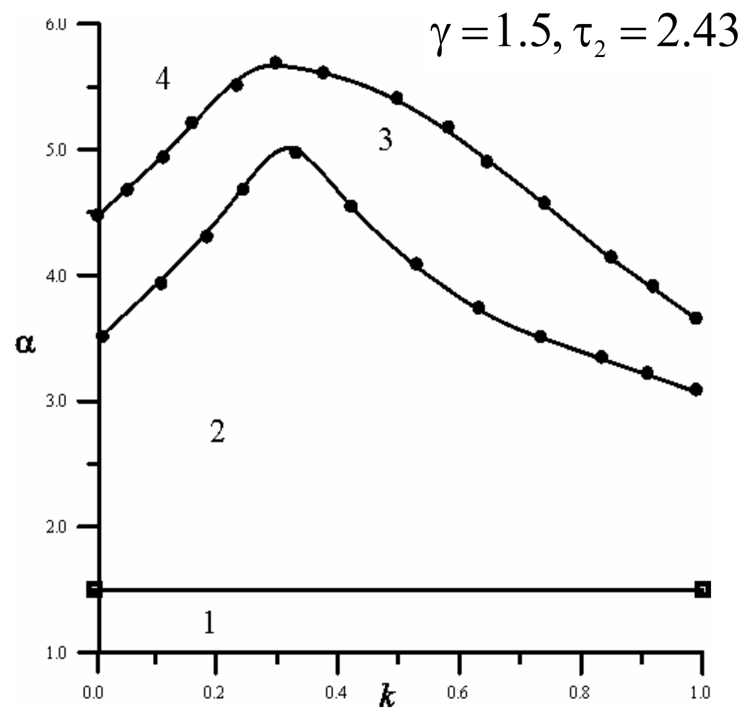
Suppressing of self-modulaton



Same for deep self modulation after period doubling bifurcation, $\alpha=4.3$

Map of dynamic regimes on *k*–a plane

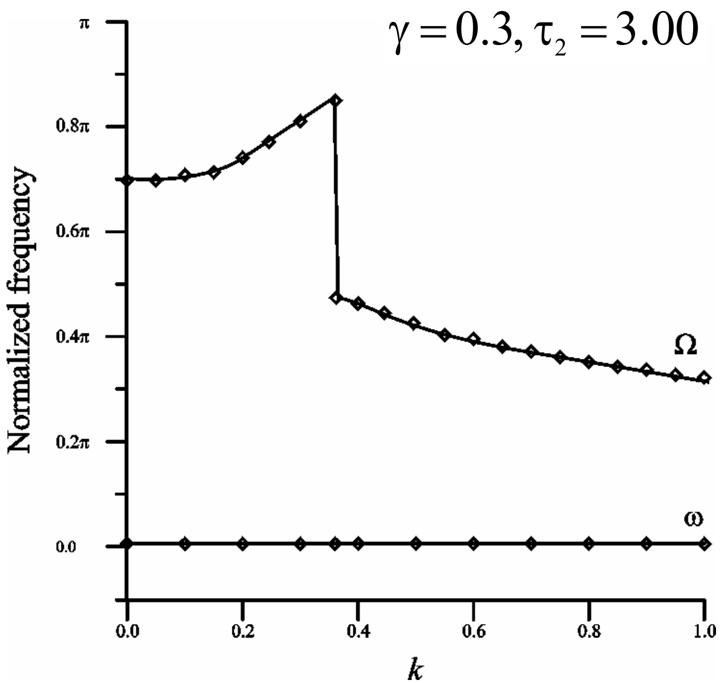


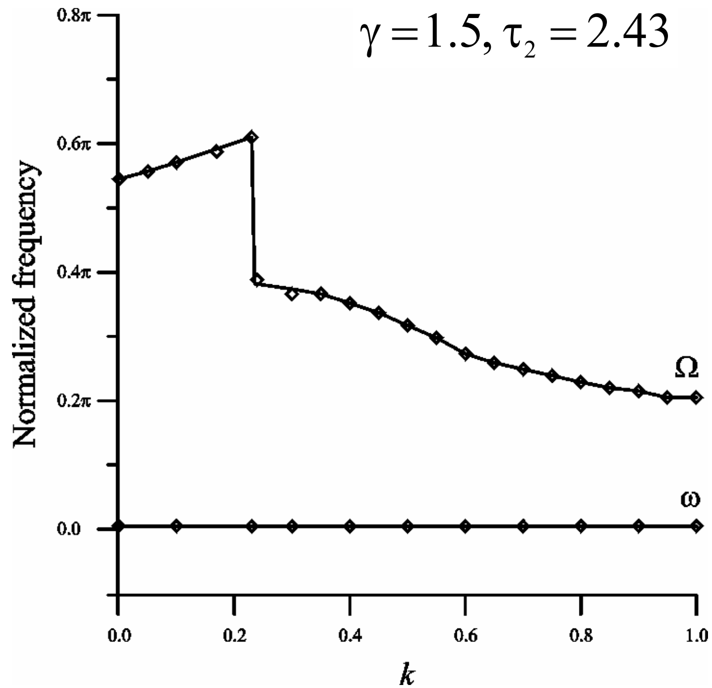


(1) — no generation, (2) — single-frequency generation, (3) — periodic self-modulation, (4) — chaos

One can see that the method works only for $k < 0.3$

This is caused by excitation of another sideband mode





Fundamental frequency does not depend on k while the modulation frequency switches to the frequency of another mode at $k \approx 0.3$

Simple 4D map model (limit $\gamma >> 1$)

$$A_{n+1} = \frac{\alpha}{\gamma}\left[(1-k)\left(1-|A_n|^2\right)A_n e^{i\psi_1} + k\left(1-|A_{n-1}|^2\right)A_{n-1}e^{i\psi_2}\right]$$

4-th order characteristic equation allows factorization in 2 second-order equations that are easy to solve analytically

$$\mu^2 + \mu\left((1-k)\left(2-\frac{\alpha}{\gamma}\right) \pm k\left(1-\frac{\alpha}{\gamma}\right)\right) - \left(k\left(2-\frac{\alpha}{\gamma}\right) \pm (1-k)\left(1-\frac{\alpha}{\gamma}\right)\right) = 0$$

$$\frac{\alpha}{\gamma} = \frac{3}{2} + \frac{1}{2(1-2k)}$$

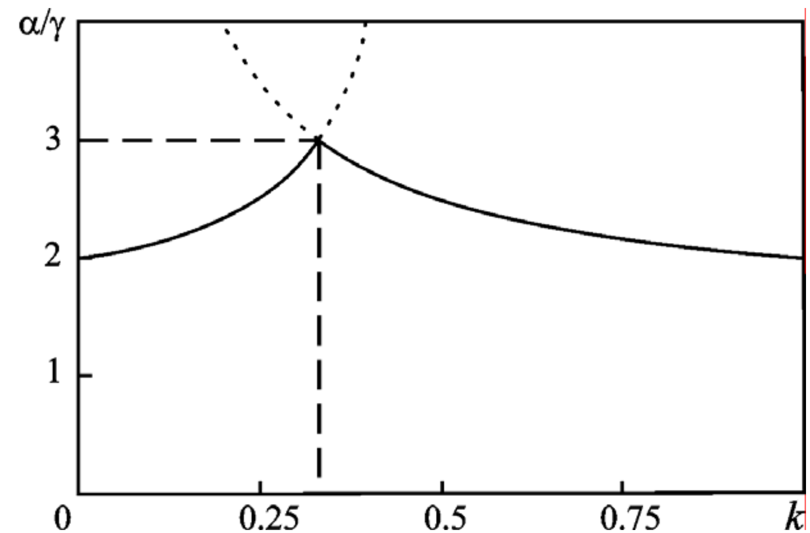
Threshold of PD bifurcation, $\mu=-1$

$$\frac{\alpha}{\gamma} = \frac{3}{2} + \frac{1}{2k}$$

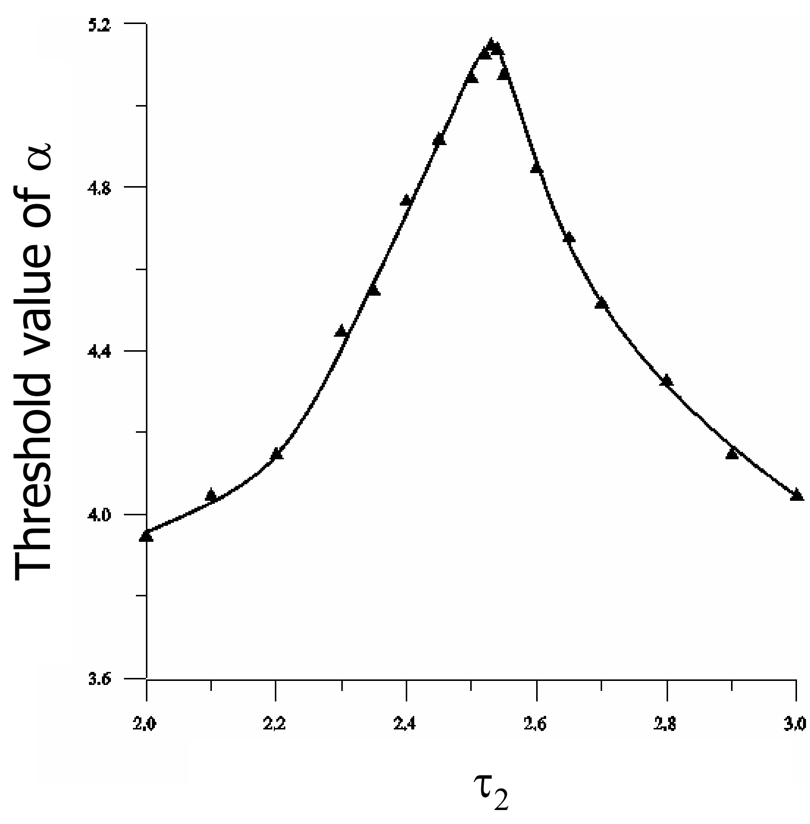
Threshold of Neimark–Sacker bifurcation, $\mu=\exp(i\theta)$

$$\cos\theta = \frac{k-1}{2k}$$

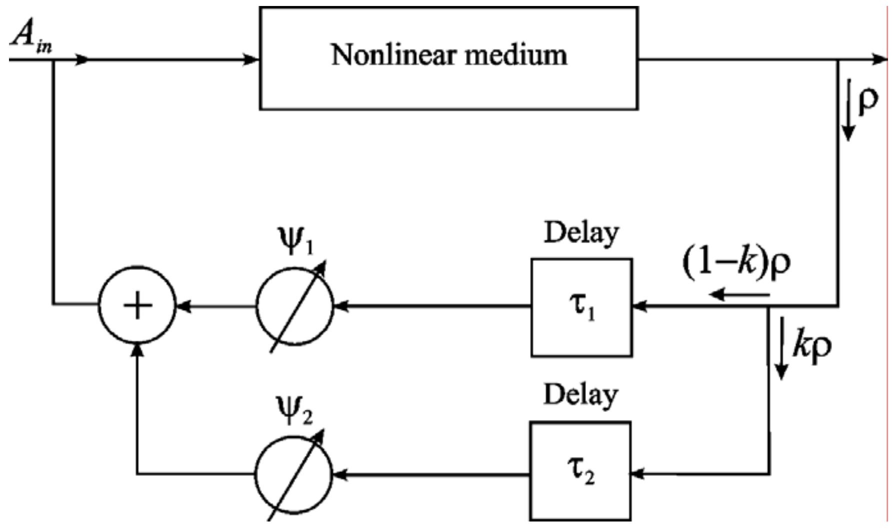
Winding number (k > 1/3)



Sensibility to mismatch of the delay parameter τ_2



Ring cavity filled with medium with cubic phase nonlinearity (Ikeda system)



$$i\left(A_t + V_g A_x\right) + \frac{\omega_0''}{2} A_{xx} + \beta |A|^2 A = 0$$

Nonlinear Schrödinger equation with delayed boundary condition

$$A(0,t) = A_0 e^{i\omega t} + (1-k)\rho e^{i\psi_1} A(L, t - T_1) + k\rho e^{i\psi_2} A(L, t - T_2)$$

Nonlinear dynamics of the single-feedback (k=0) system has been studied in details in A.A. Balyakin, N.M. Ryskin, O.S. Khavroshin, (Radiophys. Quant. Electron., 2007, to be publ.).

Modified Ikeda map (zero dispersion limit)

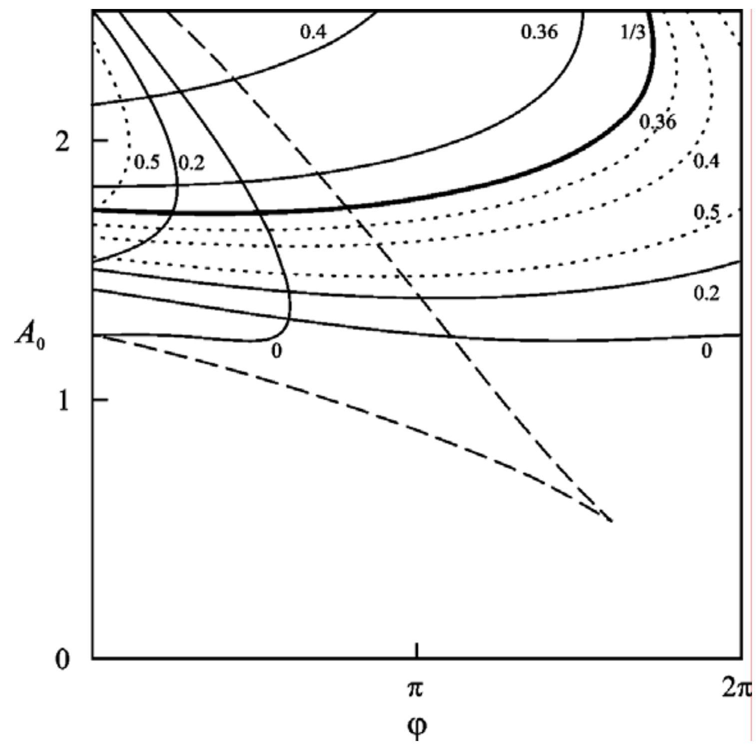
$$A_{n+1} = A_0 + (\rho - \rho_1) A_n e^{i\left(\varphi + |A_n|^2\right)} + \rho_1 A_{n-1} e^{i\left(\varphi + |A_{n-1}|^2\right)}$$

Characteristic equation

$$\mu^4 - 2\mu^2\rho\left(\mu(1-k)+k\right)\left(\cos\Phi - I\sin\Phi\right) + \rho^2\left(\mu(1-k)+k\right)^2 = 0$$

$$\Phi = \varphi + |A|^2$$

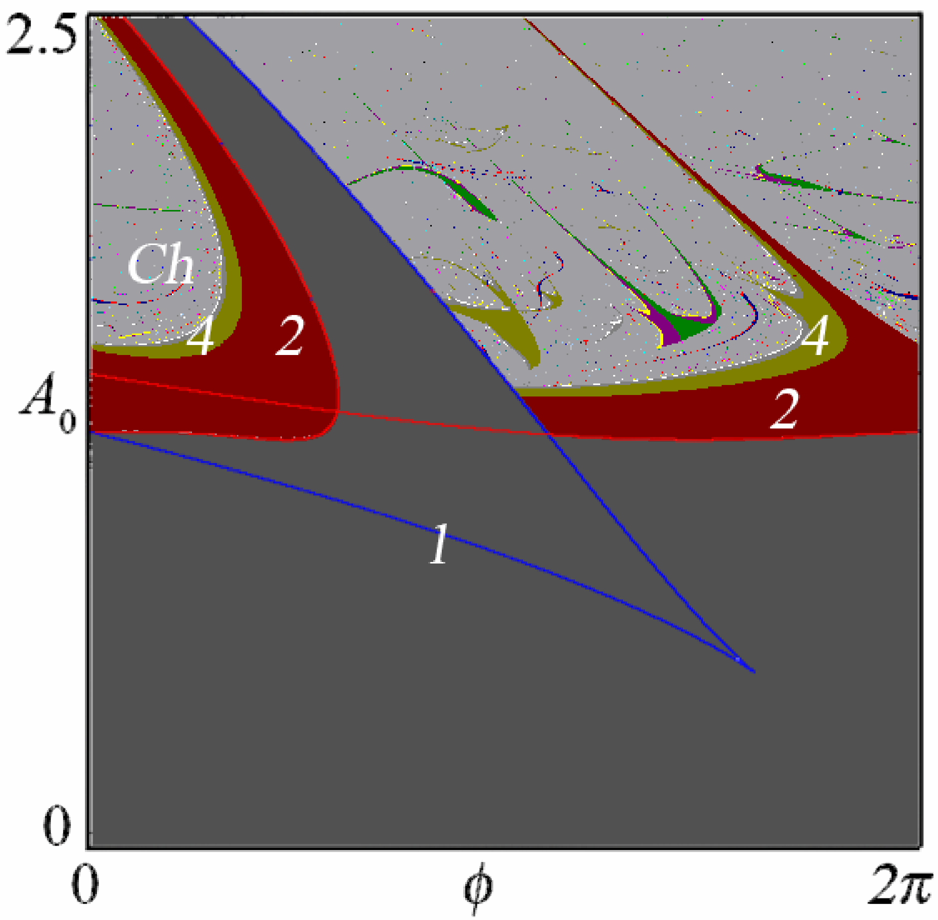
Analytical expressions for PD (solid) tangent (dashes) and Neimark–Sacker (dots) bifurcations were obtained



$$\cos\theta = \frac{k-1}{2k}$$

The same equation for the winding number ($k > 1/3$)

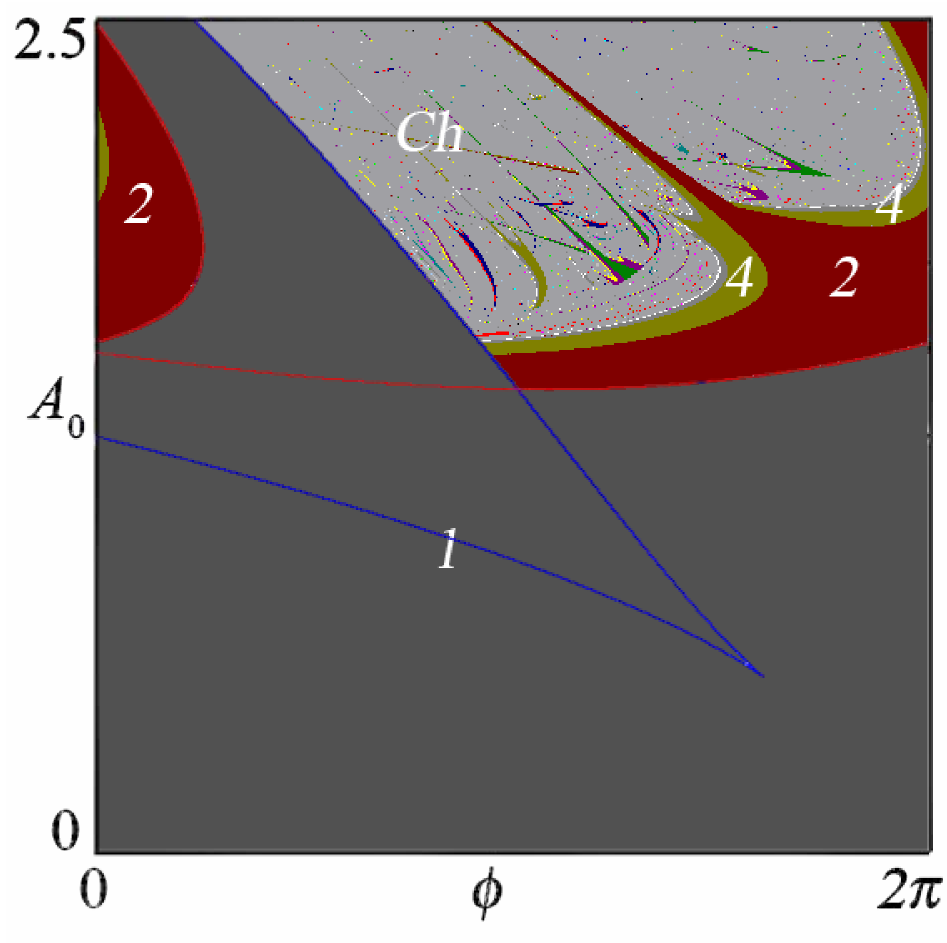
Numerical results for the modified Ikeda map



$k = 0.0$

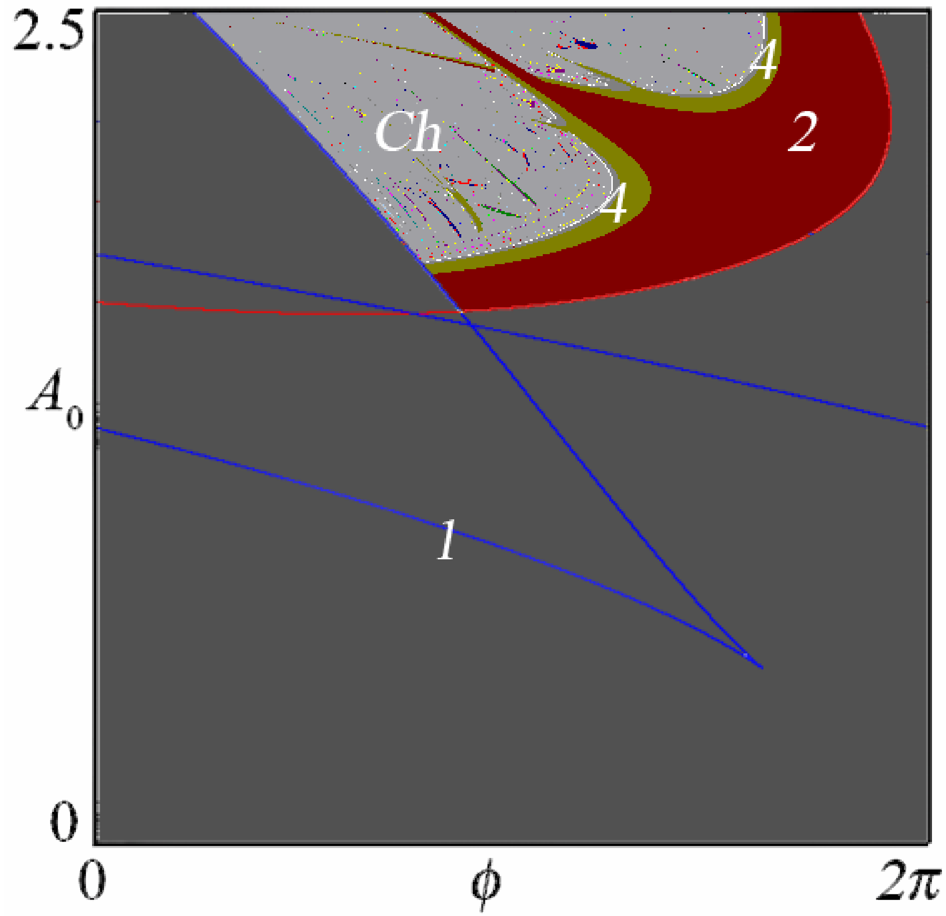
Bifurcation maps on A_0-φ plane ($\rho=0.5$). One can see excellent agreement with analytic theory. 1 — period 1 motion, 2 — period 2 motion, …, *Ch* — chaos.

Numerical results for the modified Ikeda map



$k = 0.24$

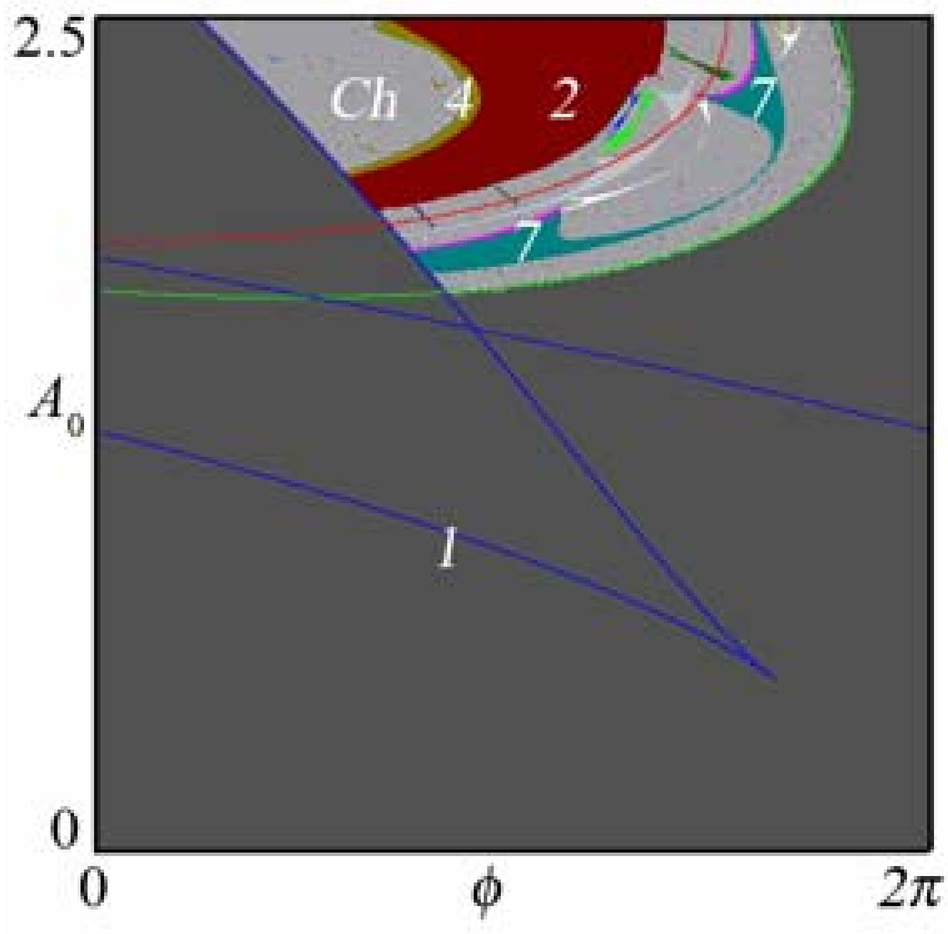
Numerical results for the modified Ikeda map



$k = 0.3$

With the increase of *k* boundary of the Ikeda instability shifts up

Numerical results for the modified Ikeda map

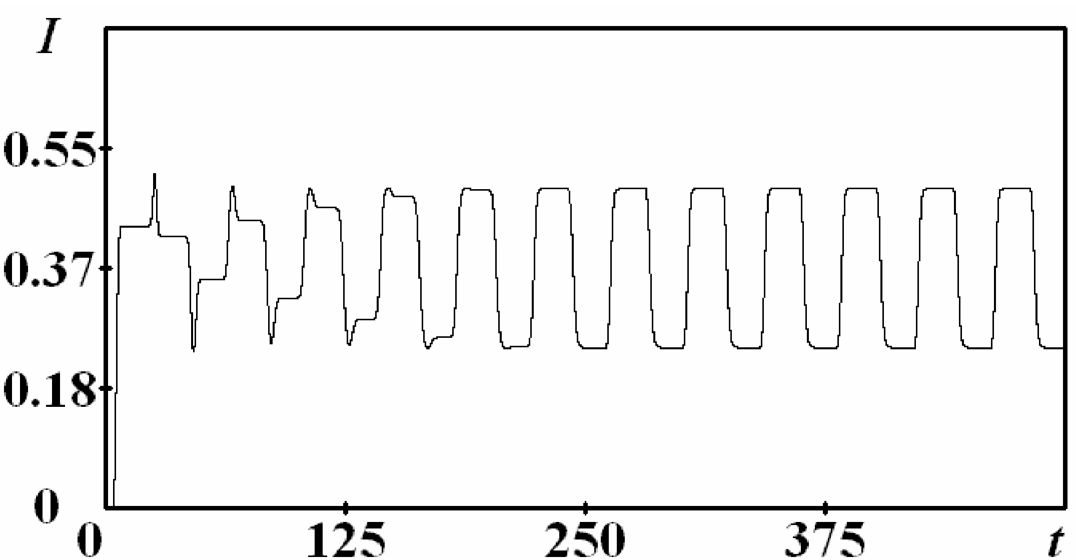


$k = 0.36$

However, for $k>1/3$ domain of quasi-periodic motion appears above the line of Neimark–Sacker bifurcation. Thus, the control is most effective for $k=1/3$.

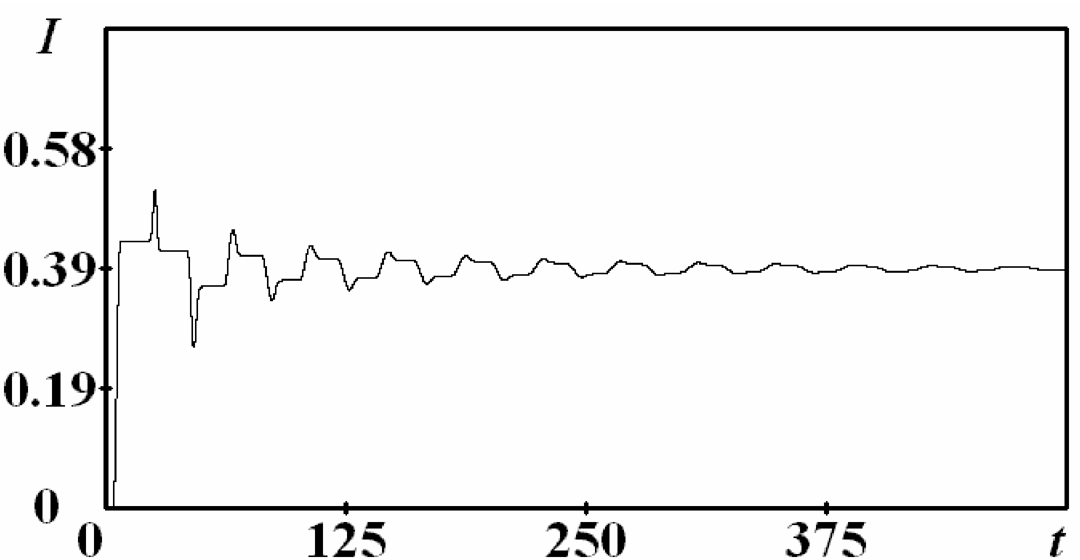
Numerical results for NLS with delayed feedback

Parameters: V=1, β=1, ω''_0=0.01, ρ=0.5, ψ_1=0, L=5, T_1=10, ω=0, A_0=0.45



$I=|A(x=L)|$

Without control (k=0) deep self-modulation with period $T_{sm}\approx 40$ is observed



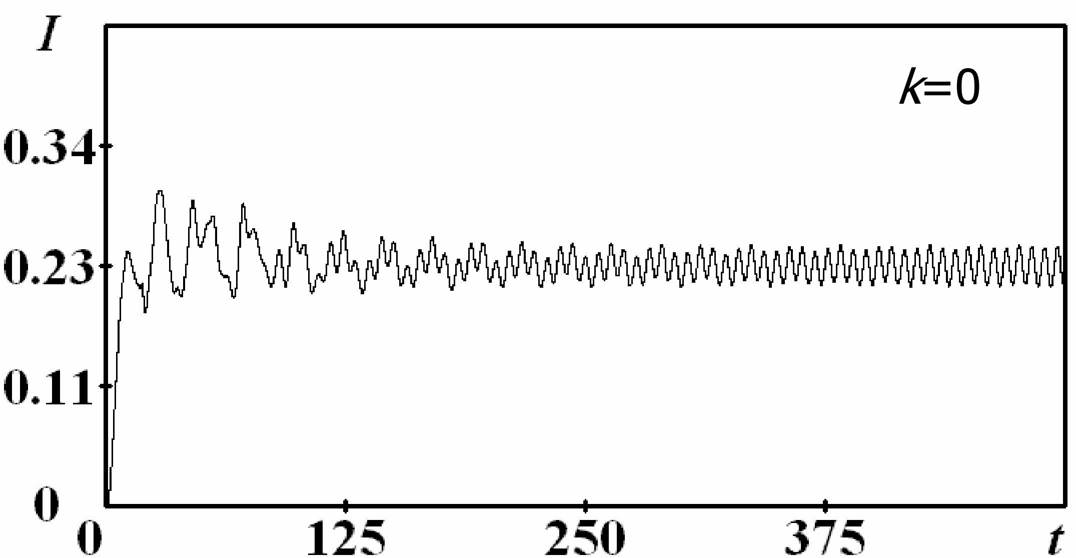
With control (k=0.2, T_2=30, ψ_2=0) we get stable single frequency oscillations

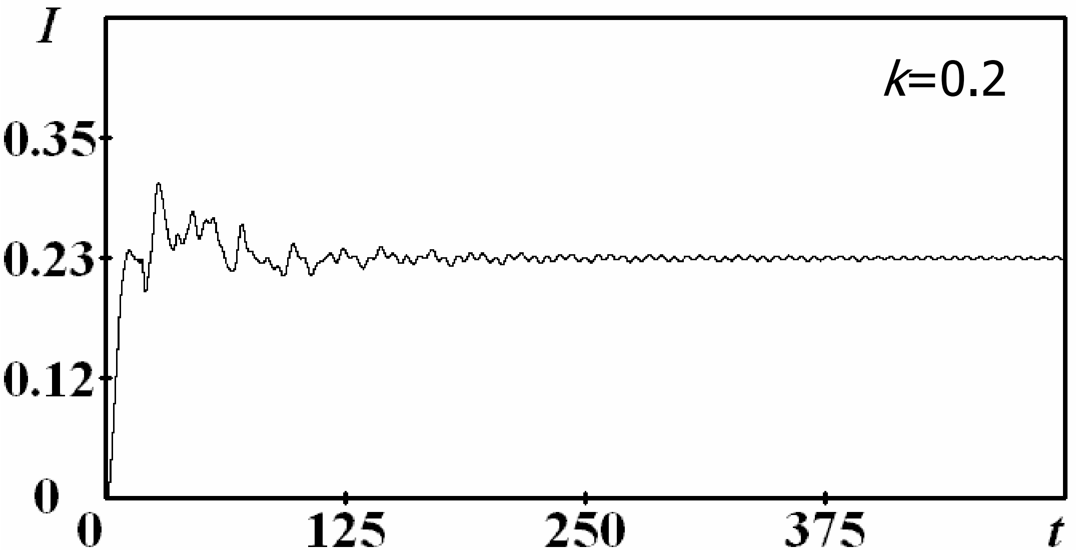
$$\psi_1 - \psi_2 - \omega\left(T_1 - T_2\right) = 2\pi n$$

$$\Omega\left(T_1 - T_2\right) = 2\pi m + \pi$$

Numerical results for NLS with delayed feedback

Same for the case of strong dispersion (ω_0''=1). $A_0 = 0.22$





Now self modulation is caused by modulation instability, not by Ikeda instability (see our paper to be publ. in Radiophys. Quant. El., 2007 for details). Completely different self-modulation period, $T_{sm}\approx 10$.

Thus we need to change the control feedback delay, T_2=5.

Summary

The proposed modification of time-delayed auto-synchronization technique for controlling chaos allows suppressing various instabilities in systems with time-delayed feedback. The method is useful to provide stable single frequency oscillations in various RF, microwave and optical devices.