Quantum Stochastic Heating of a Trapped Ion

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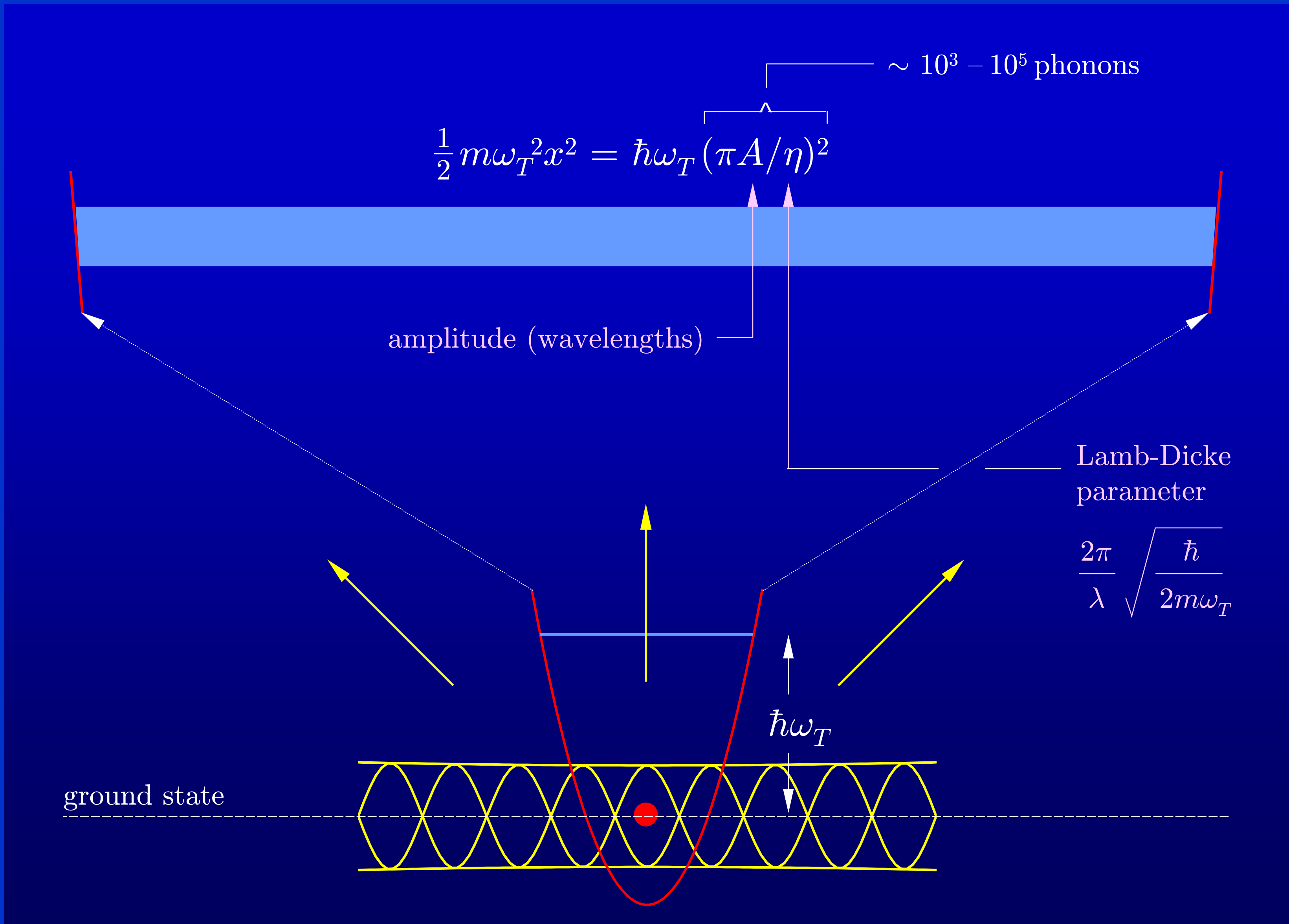

$\sim 10^3 - 10^5$ phonons
$\frac{1}{2} m\omega_T{}^2 x^2 = \hbar\omega_T (\pi A/\eta)^2$
amplitude (wavelengths)
Lamb-Dicke parameter
$\frac{2\pi}{\lambda}\sqrt{\frac{\hbar}{2m\omega_T}}$
$\hbar\omega_T$
ground state

previous work:

$$\text{Lamb-Dicke parameter } \eta \; = \begin{cases} 2\pi \dfrac{\Delta x}{\lambda} \\[2ex] 2 \dfrac{\hbar k}{\Delta p} \end{cases}$$

ground state uncertainty

$\eta \ll 1$

confinement to Lamb-Dicke regime

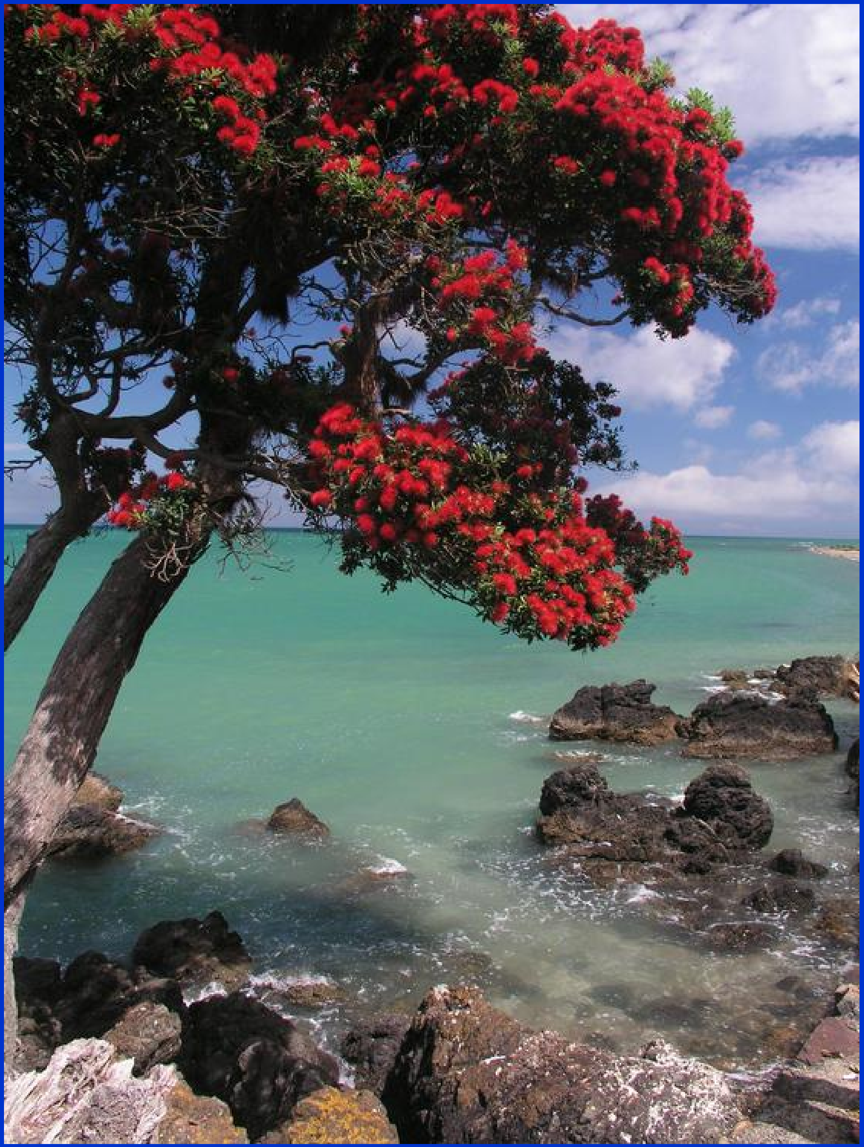
semiclassical motion in optical potential

$\dfrac{\omega_T}{\gamma} A \ll 1$

semiclassical motion in optical potential

trap frequency = atomic linewidth

Rabi frequency = 2 x atomic linewidth



<u>background</u>

semi-quantum and quantum trajectories

heating rates & stochastic dynamics

diffusion limit

large quantum jumps

semi-quantum trajectories:

conditional state:

$$|\bar{\psi}_{\mathrm{REC}}\rangle = A^{(g)}\,|g\rangle + A^{(e)}\,|e\rangle$$

nonunitary Schroedinger equation:

center of mass

$$\frac{dA^{(g)}}{dt} = -\mathcal{E}\cos[\eta(\alpha\, e^{-i\omega_T t} + \alpha^*\, e^{i\omega_T t})]\,A^{(e)}$$

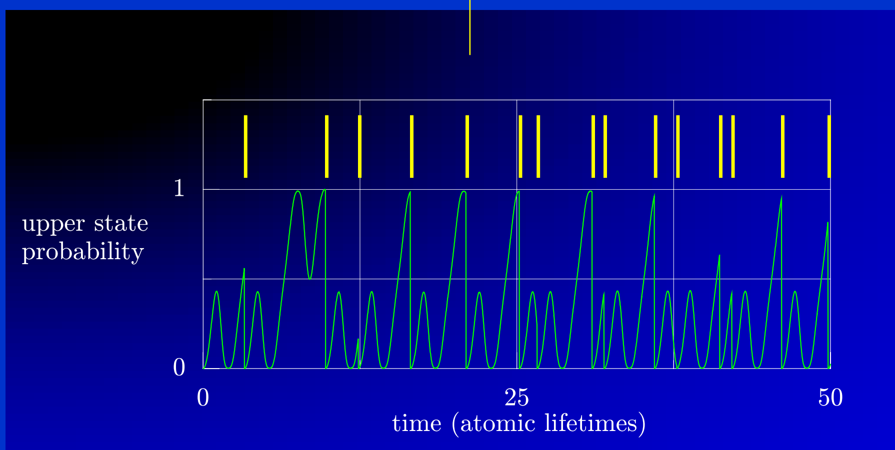
$$\frac{dA^{(e)}}{dt} = \mathcal{E}\cos[\eta(\alpha\, e^{-i\omega_T t} + \alpha^*\, e^{i\omega_T t})]\,A^{(g)} - \frac{\gamma}{2}\,A^{(e)}$$

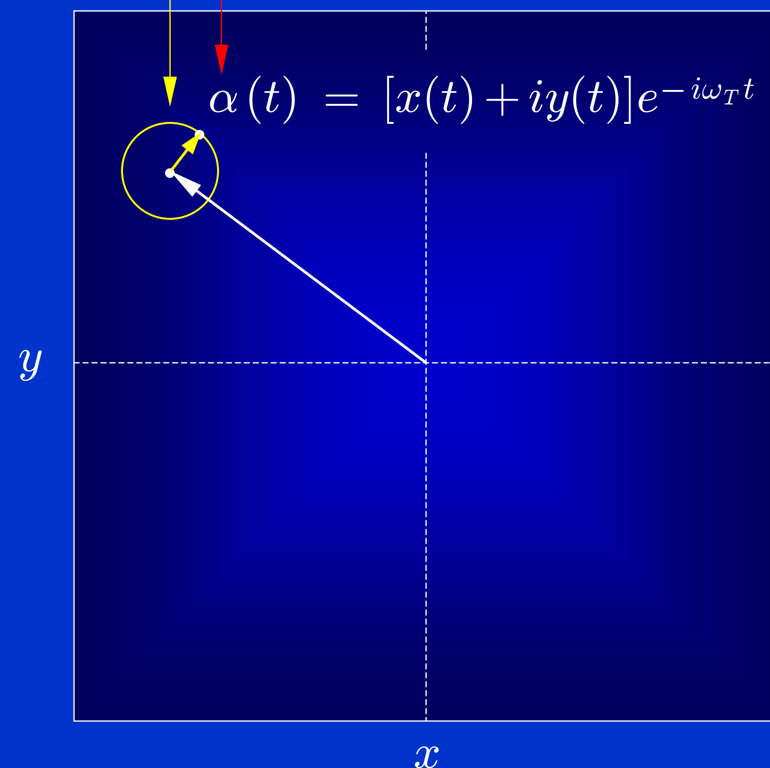
quantum jumps:

$$|\bar{\psi}_{\mathrm{REC}}\rangle \rightarrow |g\rangle \qquad \text{at rate} \qquad \gamma \times \frac{|A^{(e)}|^2}{|A^{(g)}|^2 + |A^{(e)}|^2}$$

momentum kick: $i\eta \sin\theta \cos\phi \, e^{i\omega_T t}$

center of mass





quantum trajectories:

conditional state:

$$|\,\bar{\psi}_{\rm REC}\rangle \;\; = |g\rangle|\,\bar{\psi}^{(g)}_{\rm REC}\rangle \;\; + |e\rangle|\,\bar{\psi}^{(e)}_{\rm REC}\rangle$$

nonunitary Schroedinger equation:

center of mass

$$\frac{d\,|\,\bar{\psi}^{(g)}_{\rm REC}\rangle}{dt} = -\mathcal{E}\cos[\eta(\hat{a}e^{-i\omega_T t} + \hat{a}^\dagger e^{i\omega_T t})]\,|\,\bar{\psi}^{(e)}_{\rm REC}\rangle$$

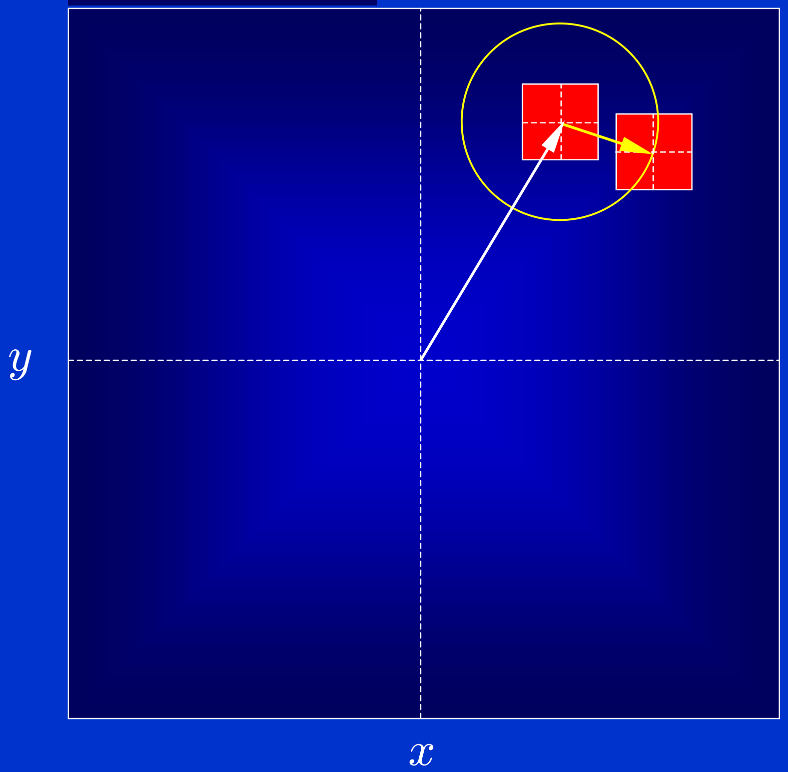
$$\frac{d\,|\,\bar{\psi}^{(e)}_{\rm REC}\rangle}{dt} = \mathcal{E}\cos[\eta(\hat{a}e^{-i\omega_T t} + \hat{a}^\dagger e^{i\omega_T t})]\,|\,\bar{\psi}^{(g)}_{\rm REC}\rangle \; - \; \frac{\gamma}{2}\,|\,\bar{\psi}^{(e)}_{\rm REC}\rangle$$

quantum jumps:

$$|\,\bar{\psi}_{\rm REC}\rangle \to |g\rangle \exp(ik\hat{x})|\bar{\psi}^{(e)}_{\rm REC}\rangle \quad \text{at rate} \quad \gamma \times \frac{\langle\bar{\psi}^{(e)}_{\rm REC}\,|\,\bar{\psi}^{(e)}_{\rm REC}\rangle}{\langle\bar{\psi}^{(g)}_{\rm REC}\,|\,\bar{\psi}^{(g)}_{\rm REC}\rangle + \langle\bar{\psi}^{(e)}_{\rm REC}\,|\,\bar{\psi}^{(e)}_{\rm REC}\rangle}$$

center of mass

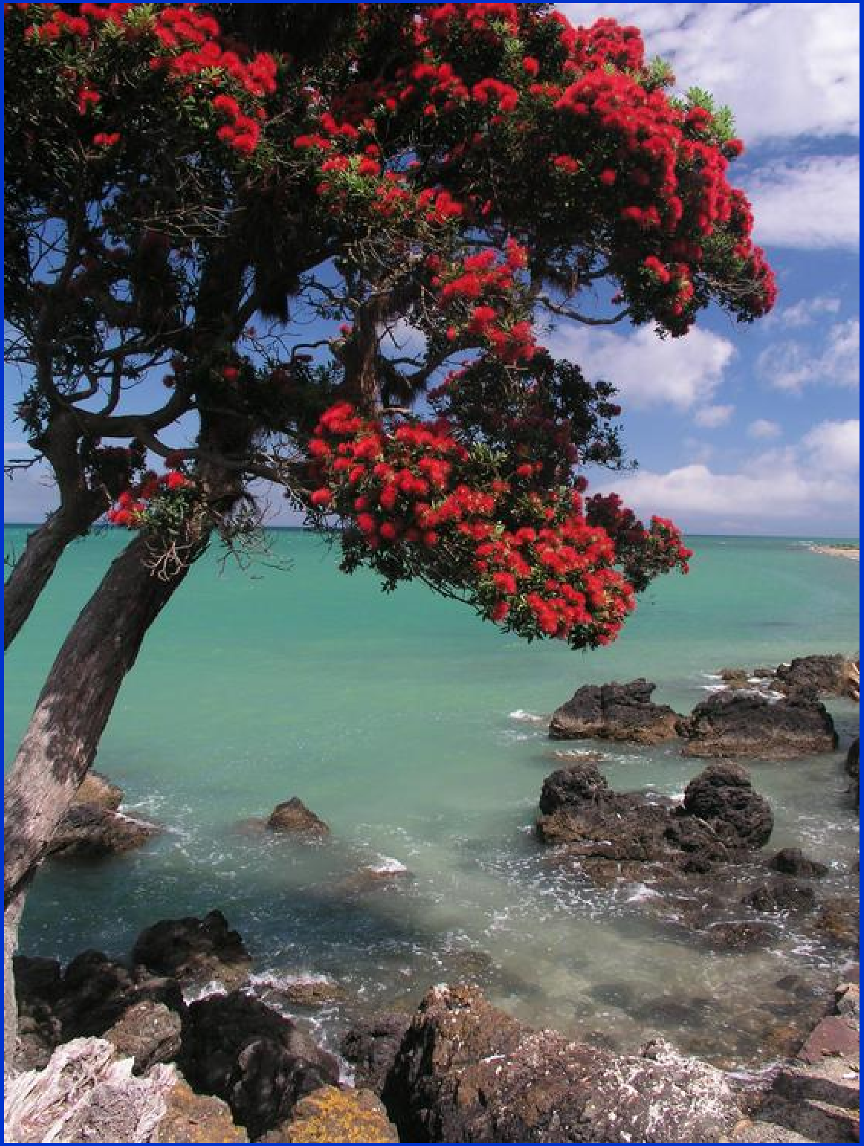
$$\hat{a} = \alpha(t) + \hat{b}$$



momentum kick:

$$\hat{D}\left(i\eta\,\sin\theta\cos\phi\,e^{i\omega_T t}\right)$$

$$\alpha(t) \rightarrow \alpha(t) + i\eta\,\sin\theta\,\cos\phi\,e^{i\omega_T t}$$



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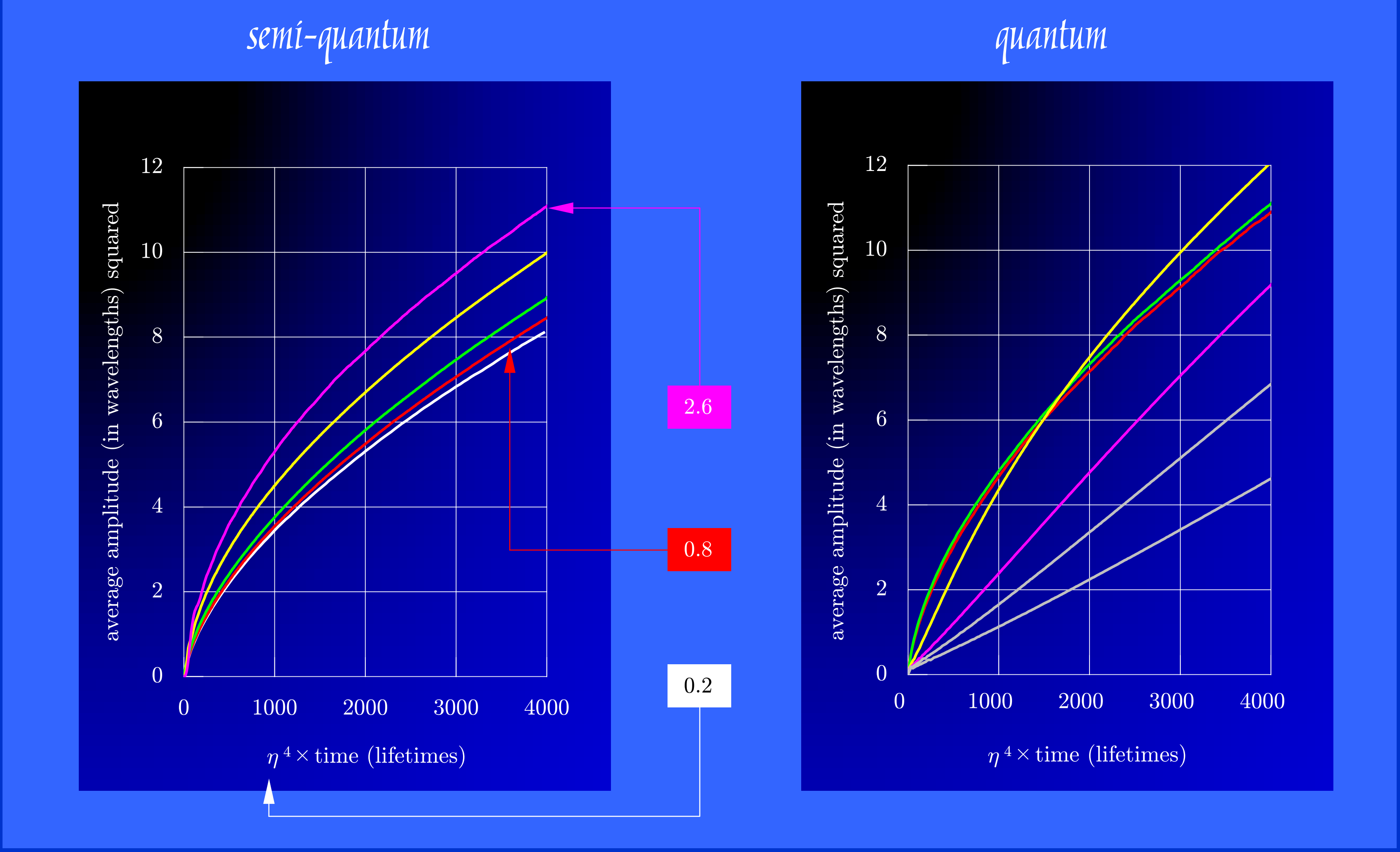
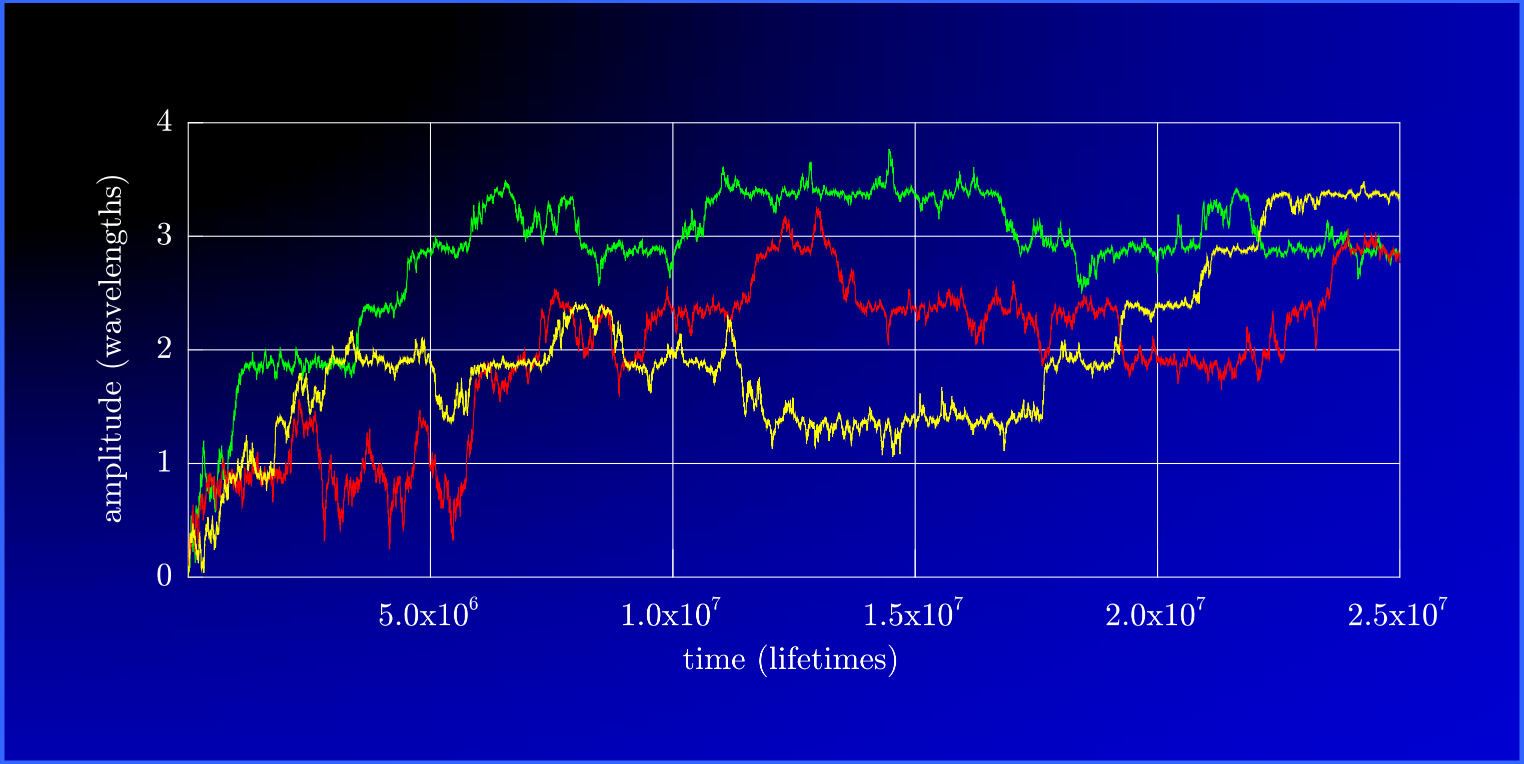
heating rates:

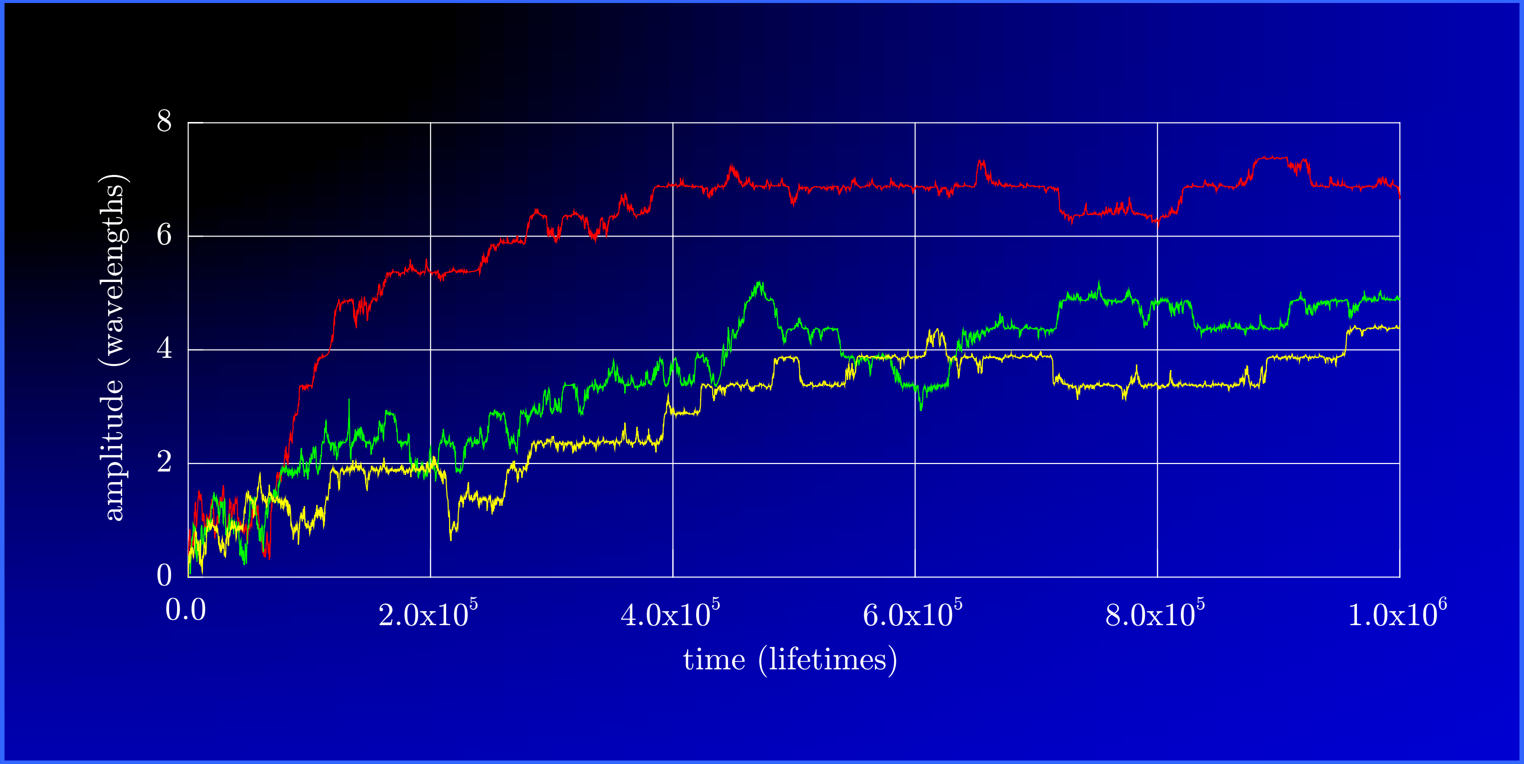


sample simulations:

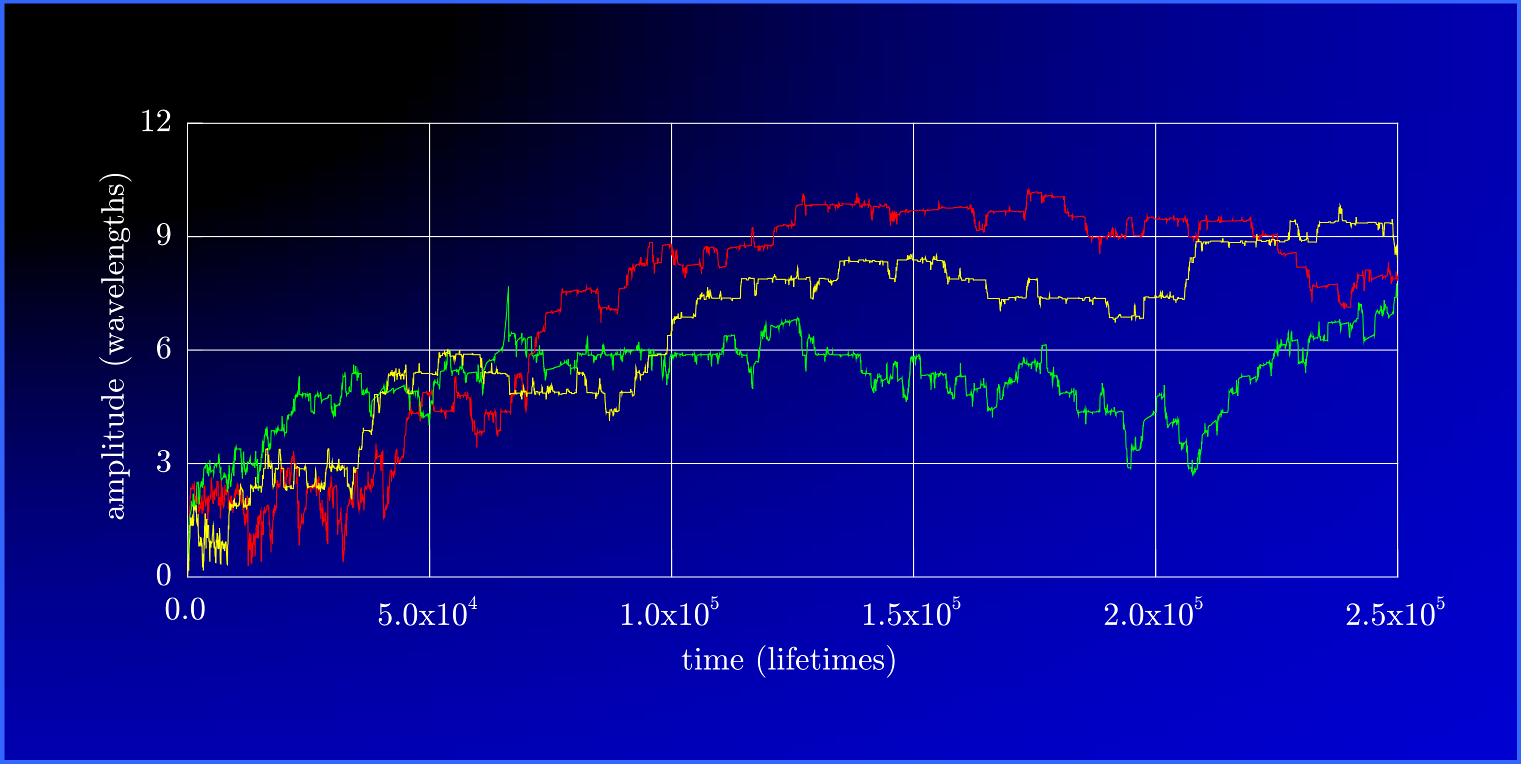
Lamb-Dicke parameter 0.11



Lamb-Dicke parameter 0.33

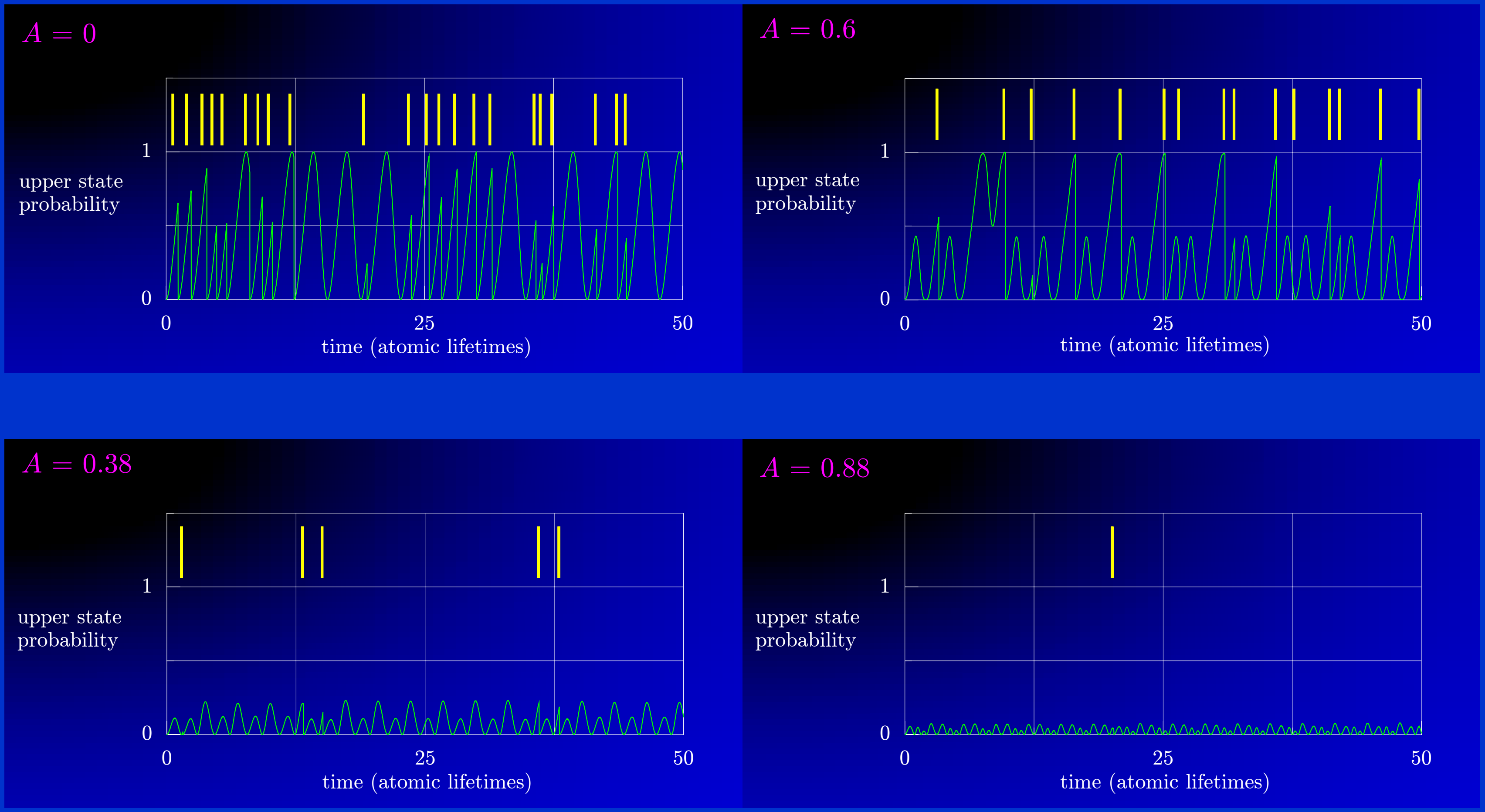


Lamb-Dicke parameter 0.79



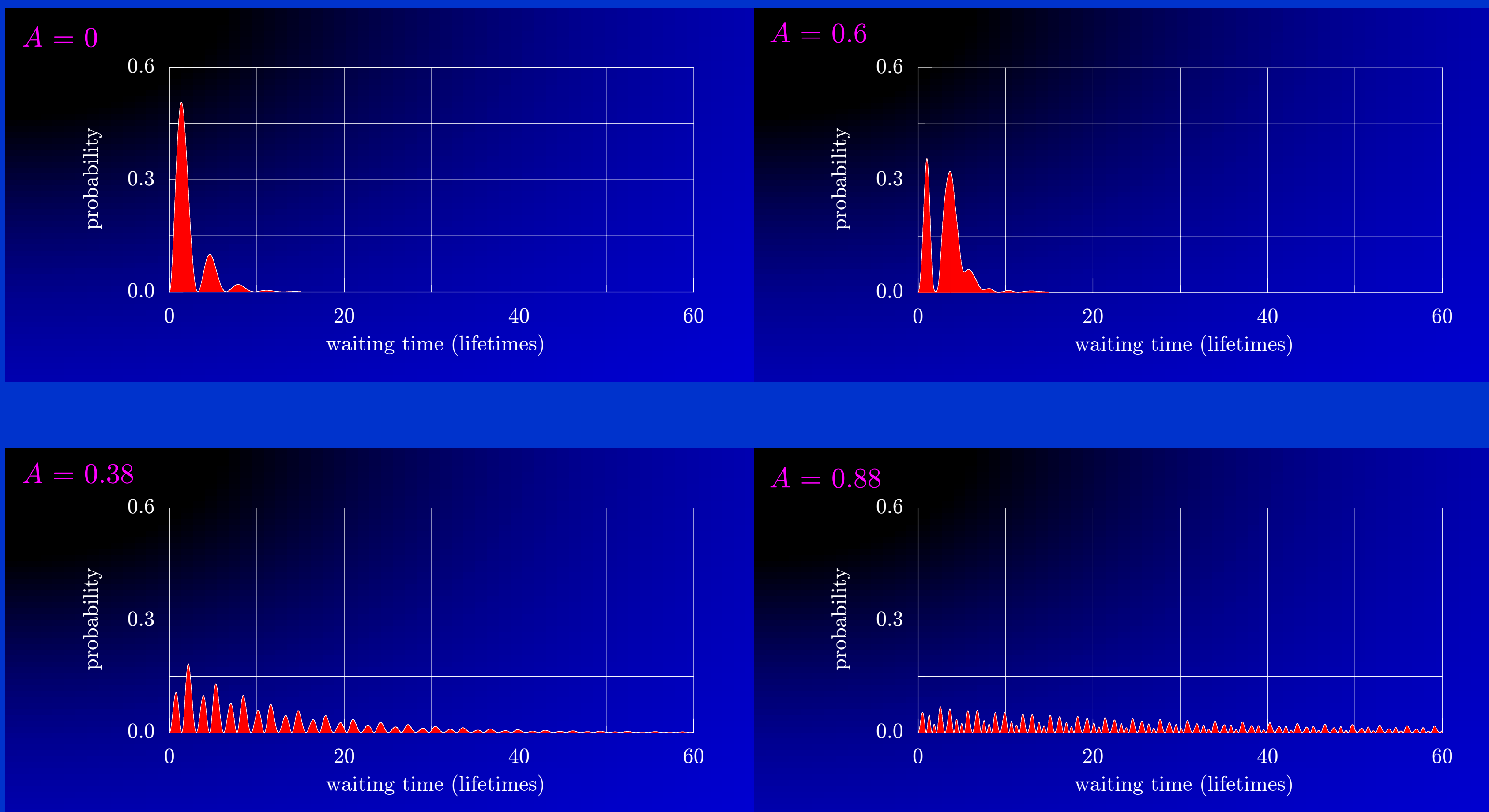
Rabi frequency modulation:

time averaged driving field $\cos[kx(t)] \longrightarrow J_0(2\pi A)$

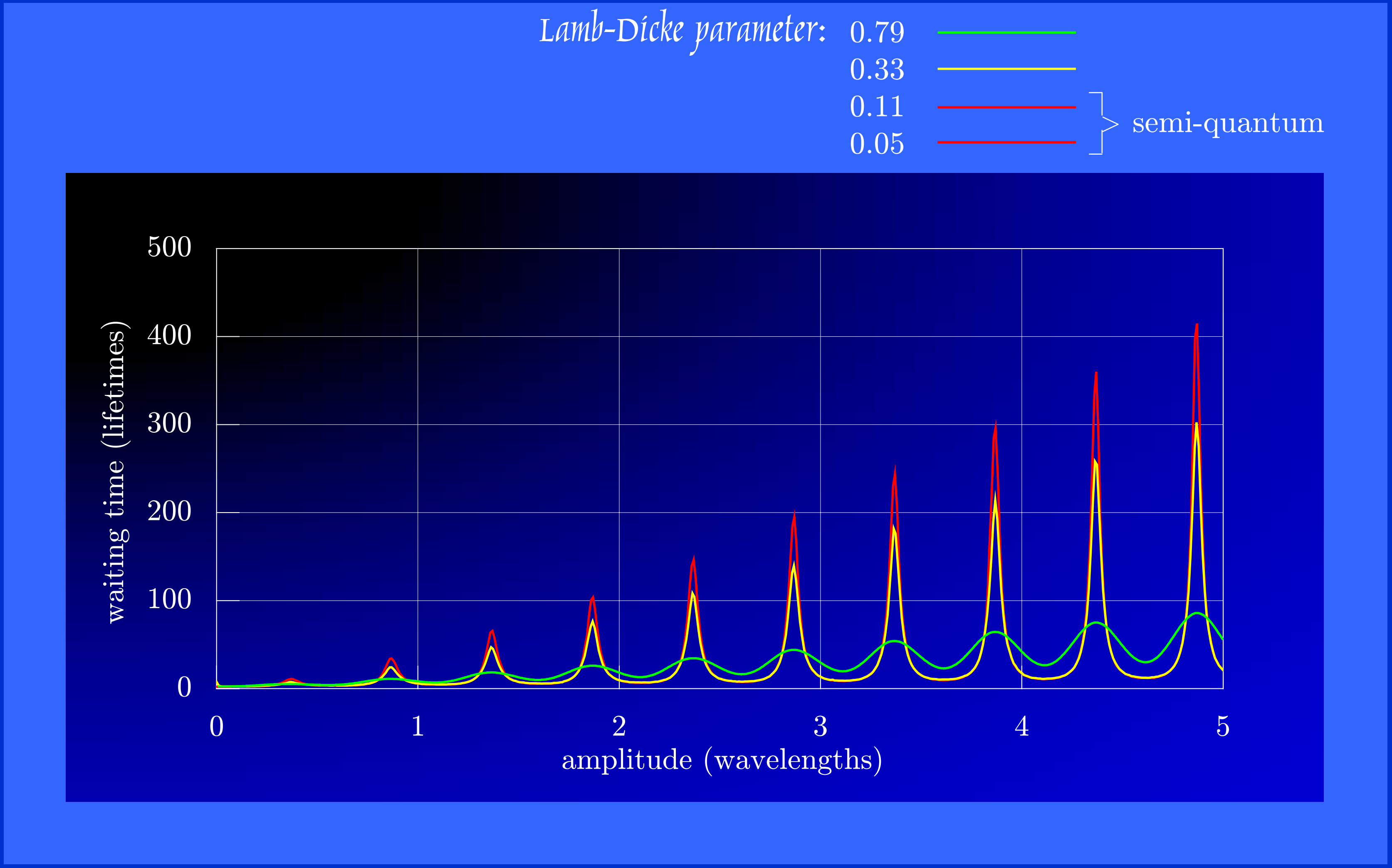


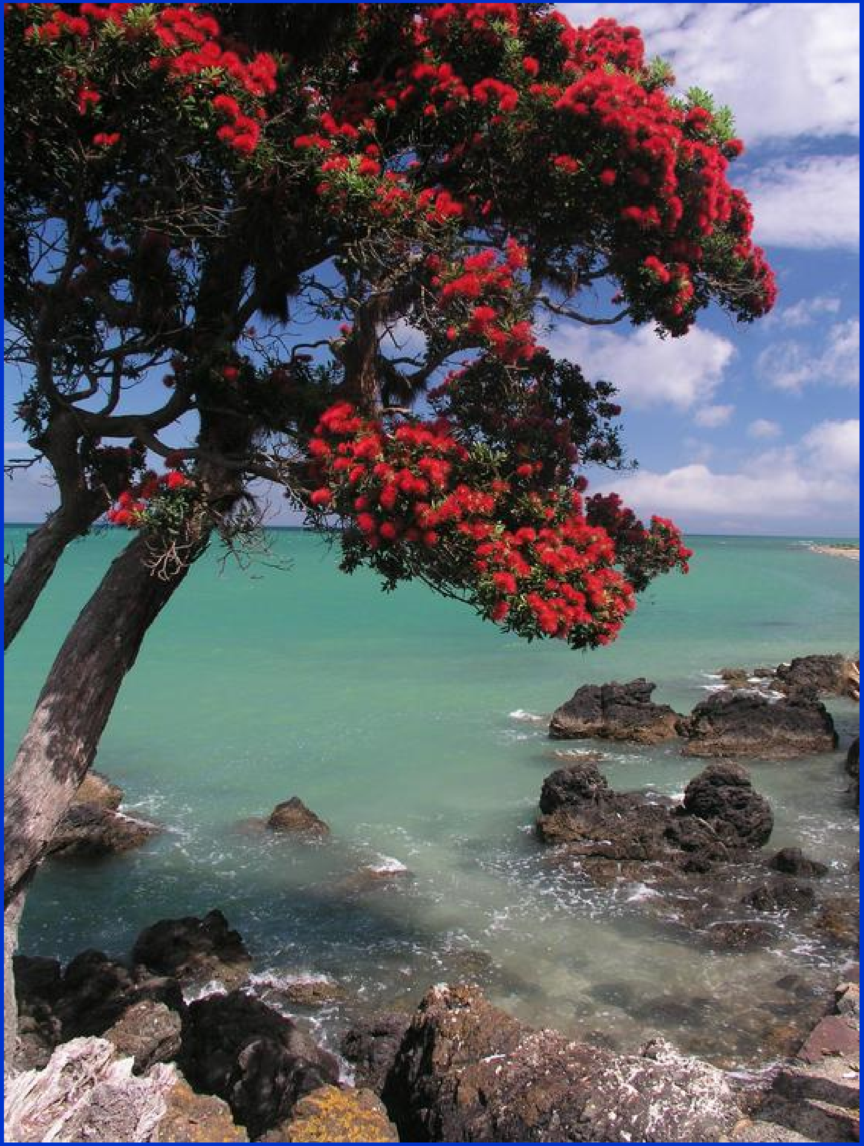
waiting time distribution:

$$i\hbar \frac{d\,|\,\bar{\psi}_{\mathrm{REC}}\rangle}{d\tau} = \hat{H}_B(\tau)\,|\,\bar{\psi}_{\mathrm{REC}}\rangle \longrightarrow W(\tau) = \gamma\,\langle\bar{\psi}_{\mathrm{REC}}(\tau)\,|\,\hat{\sigma}_+\hat{\sigma}_-\,|\,\bar{\psi}_{\mathrm{REC}}(\tau)\rangle$$



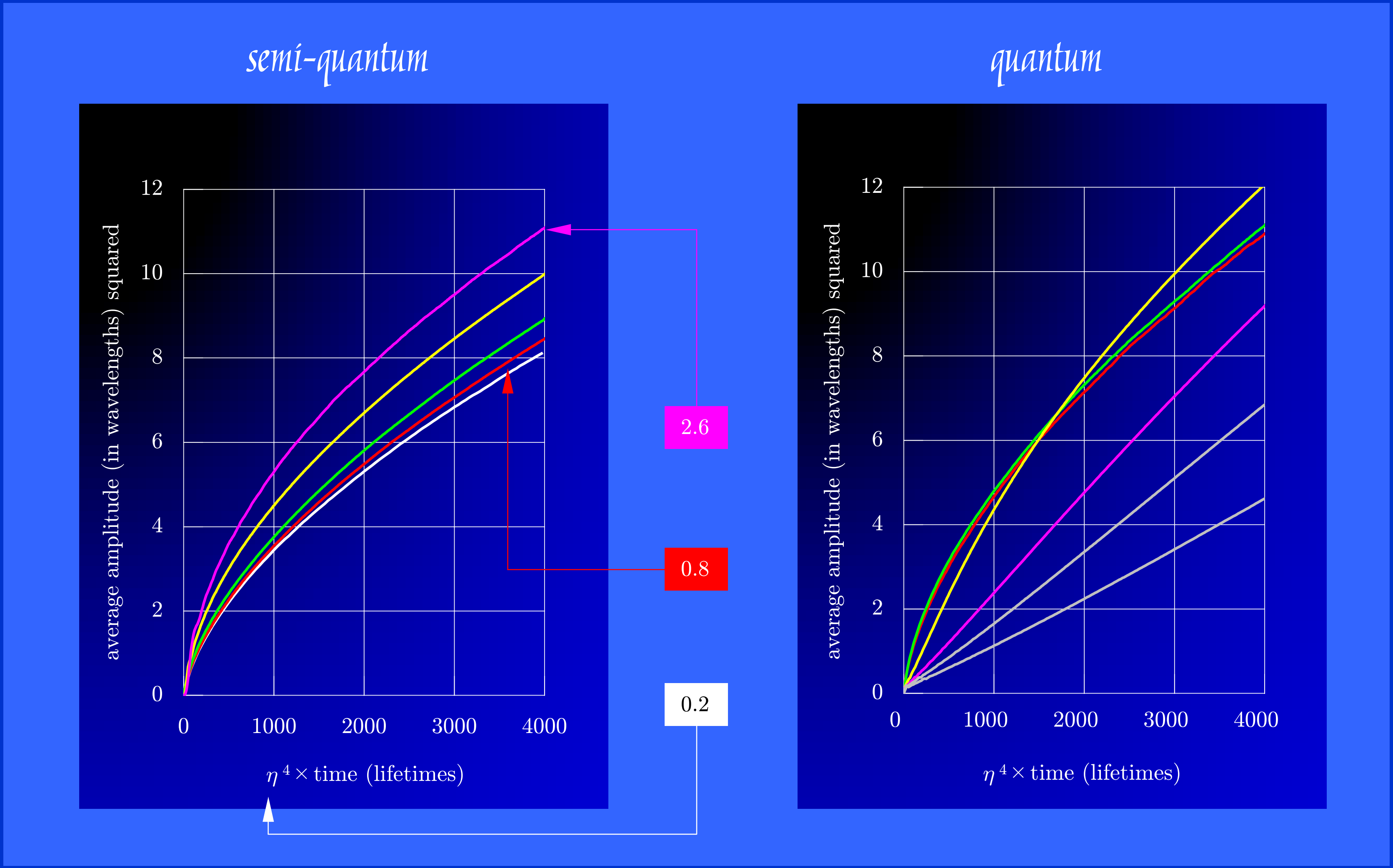
mean waiting time:





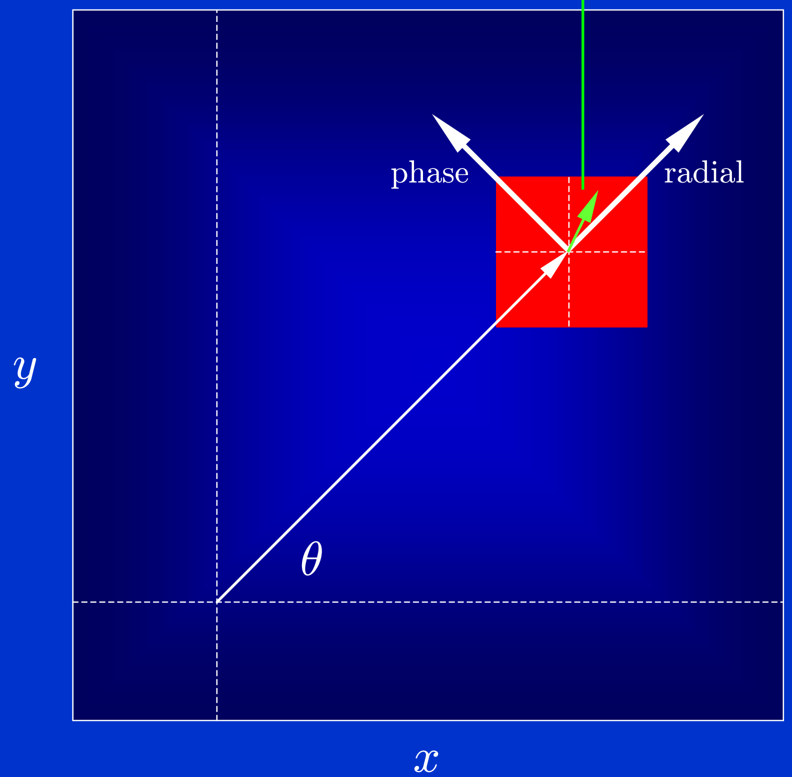
- background
- semi-quantum and quantum trajectories
- heating rates & stochastic dynamics
- diffusion limit
- large quantum jumps

heating rates:



diffusion limit:

single quantum event of order η



$$\Delta = \Delta_{\mathrm{radial}} + i\Delta_{\mathrm{phase}}$$

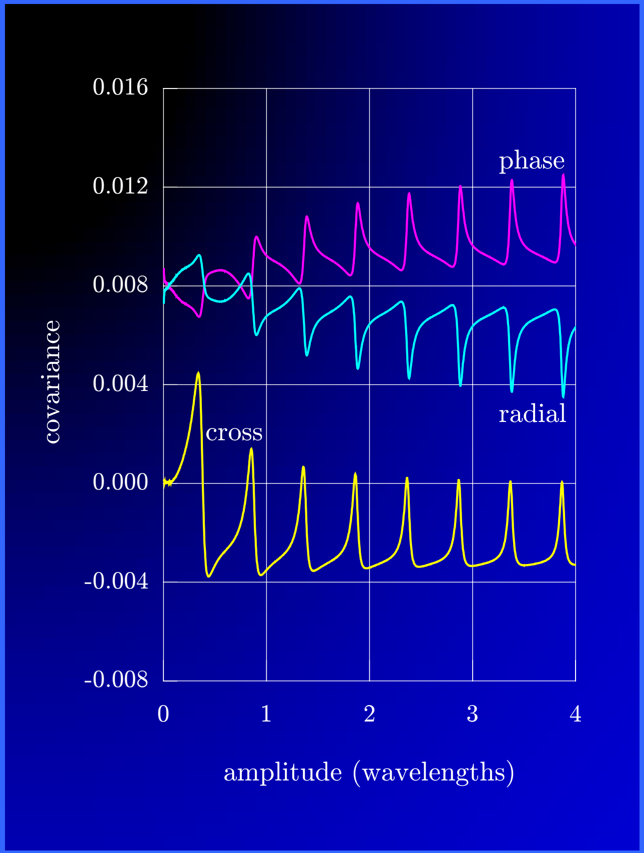
$$BB^T = \begin{pmatrix} \overline{\Delta_{\mathrm{radial}}\Delta_{\mathrm{radial}}} & \overline{\Delta_{\mathrm{radial}}\Delta_{\mathrm{phase}}} \\ \overline{\Delta_{\mathrm{radial}}\Delta_{\mathrm{phase}}} & \overline{\Delta_{\mathrm{phase}}\Delta_{\mathrm{phase}}} \end{pmatrix}$$

$$\frac{\eta}{\pi}\begin{pmatrix} dx \\ dy \end{pmatrix} = \frac{\eta}{\pi}\, R(\theta)\, \frac{B(A)}{\sqrt{\tau(A)}} \begin{pmatrix} dW_x \\ dW_y \end{pmatrix}$$

amplitude-dependent diffusion

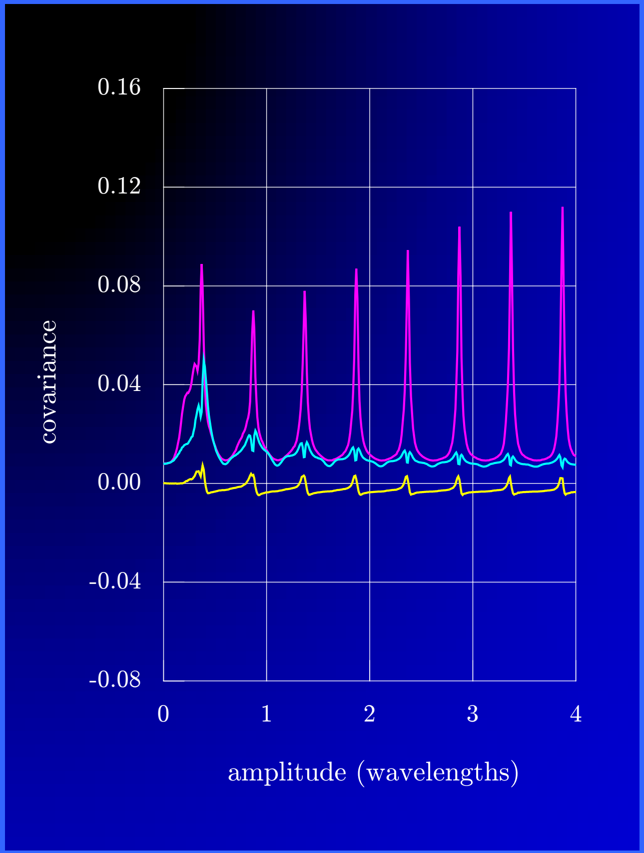
covariance matrix:

semi-quantum



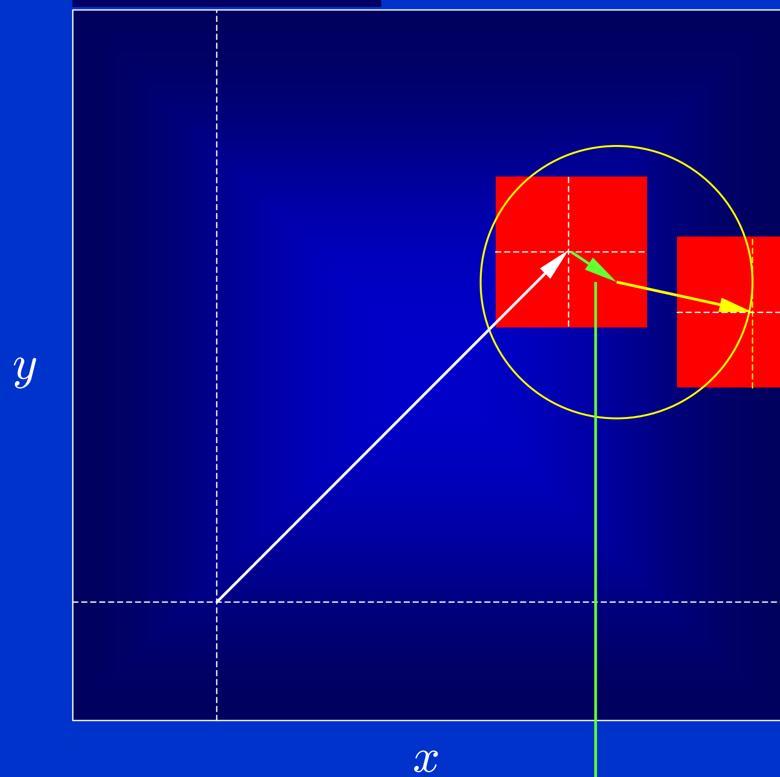
Lamb-Dicke parameter: 0.2

quantum



center of mass

$$\hat{a} = \alpha(t) + \hat{b}$$



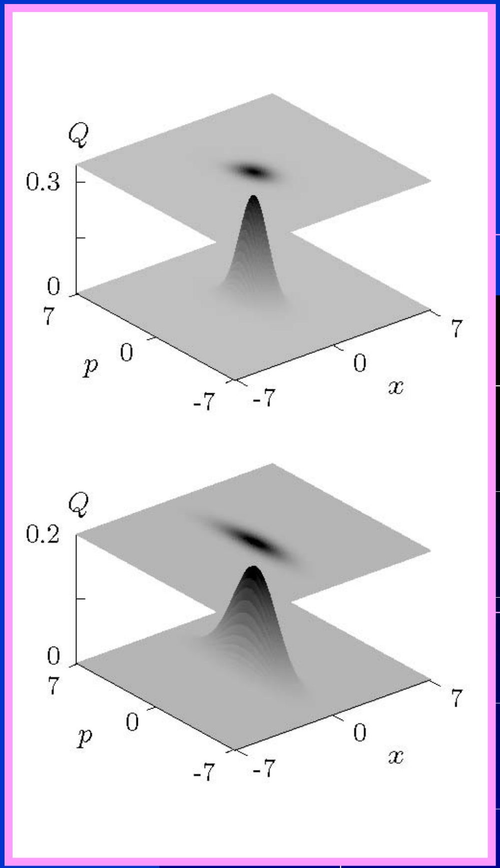
momentum kick

plus

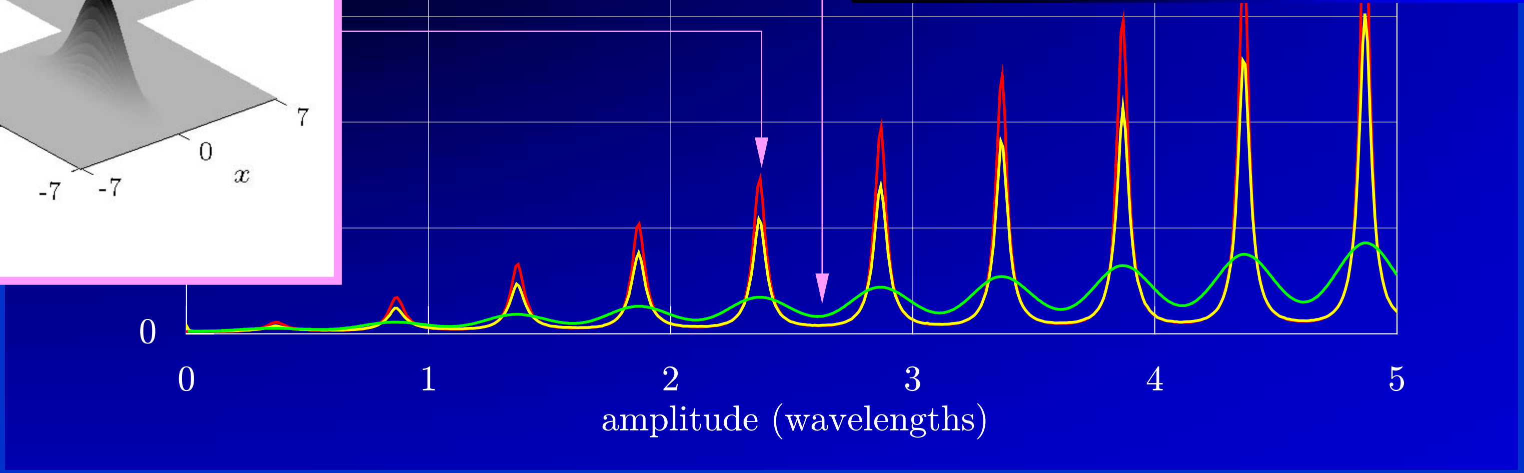
stochastic dipole displacement:

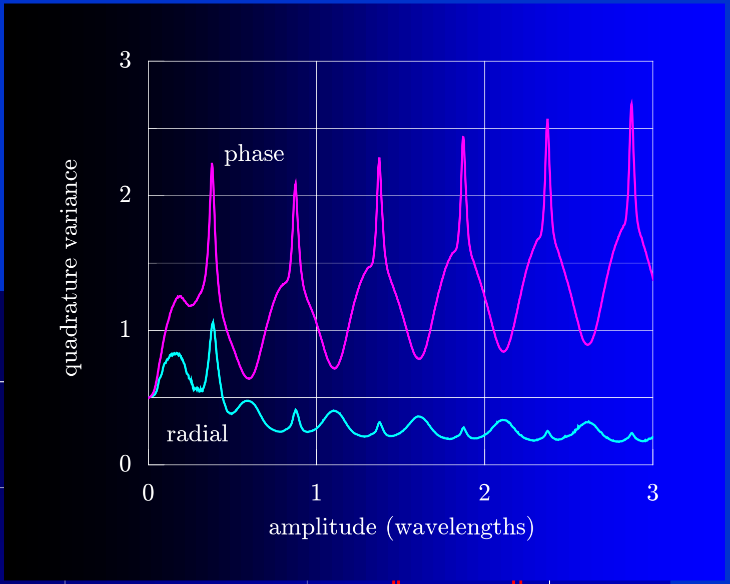
$$\alpha(t) \rightarrow \alpha(t) + i\eta \sin\theta \cos\phi \, e^{i\omega_T t} + \delta\alpha$$

quantum measurement and squeezing:

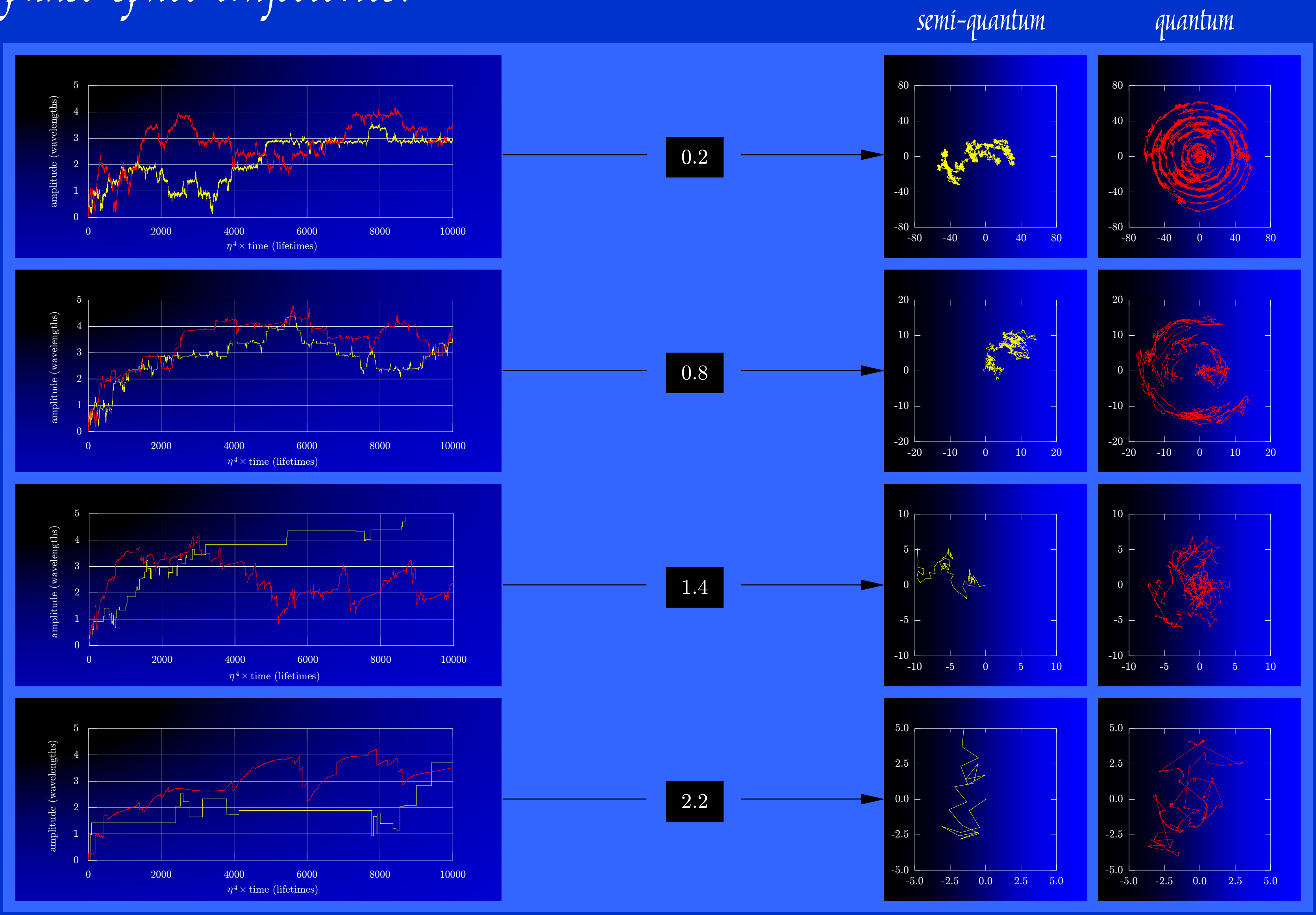


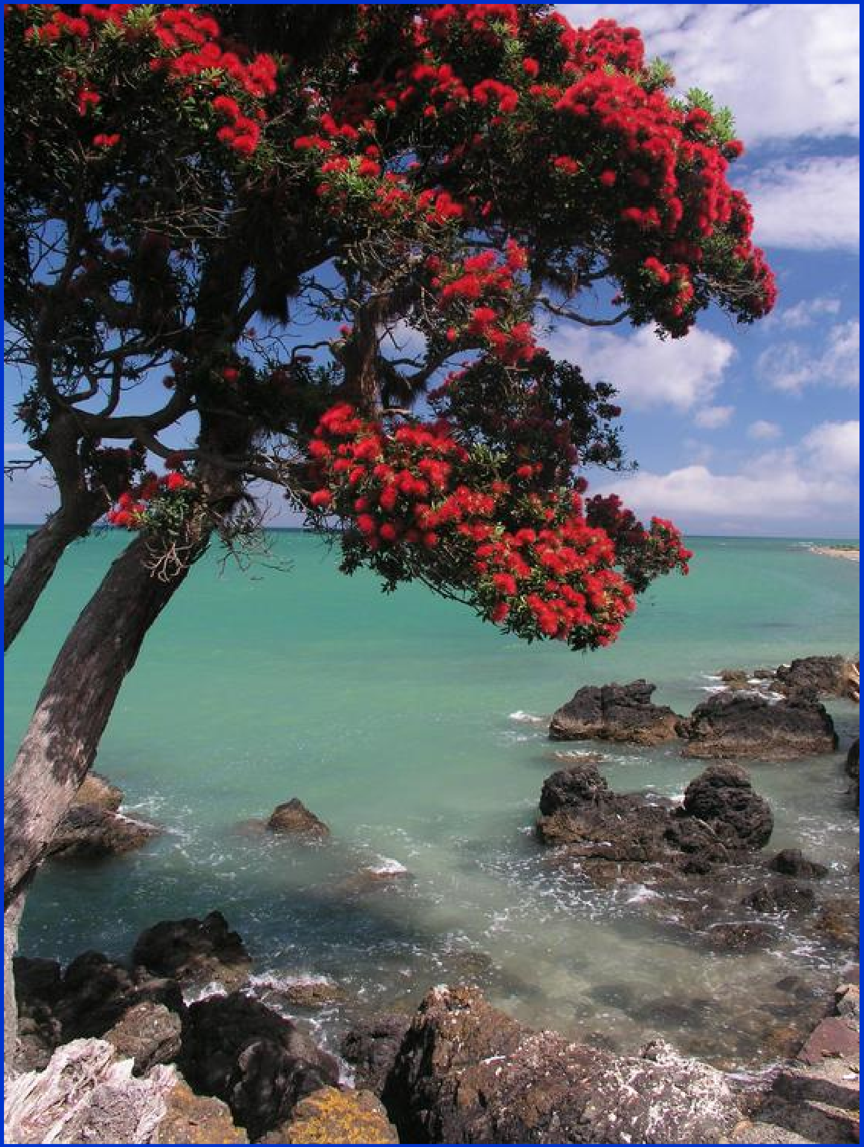
Lamb-Dicke parameter: 0.2





phase-space trajectories:





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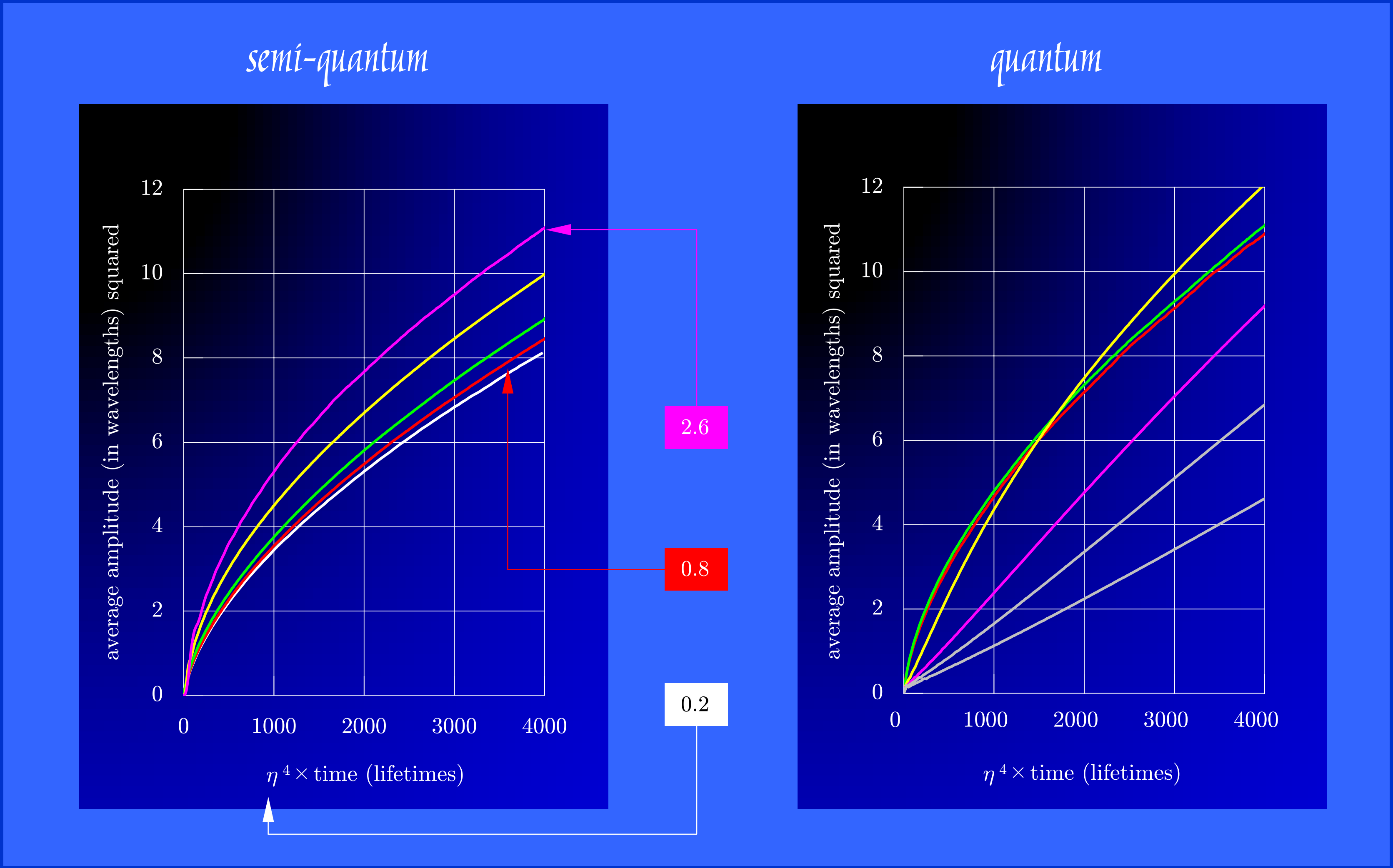
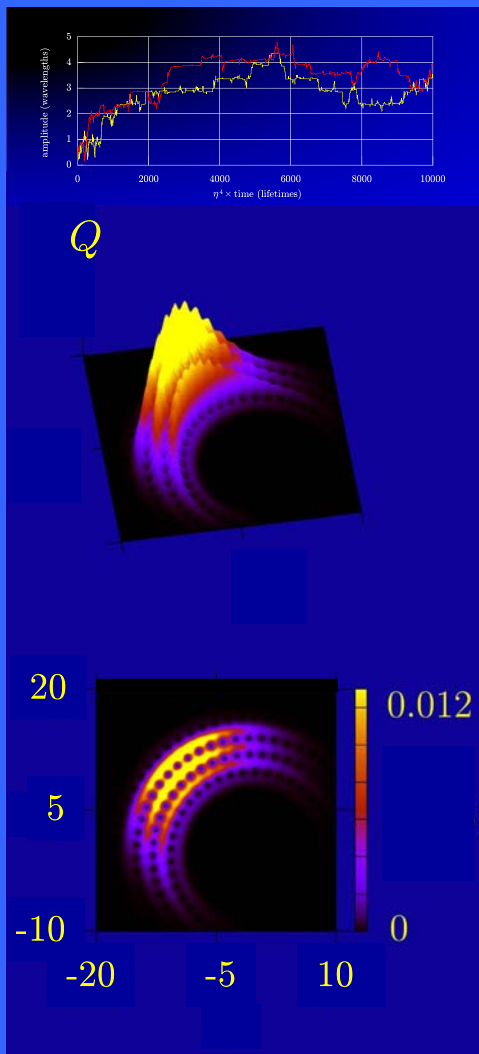
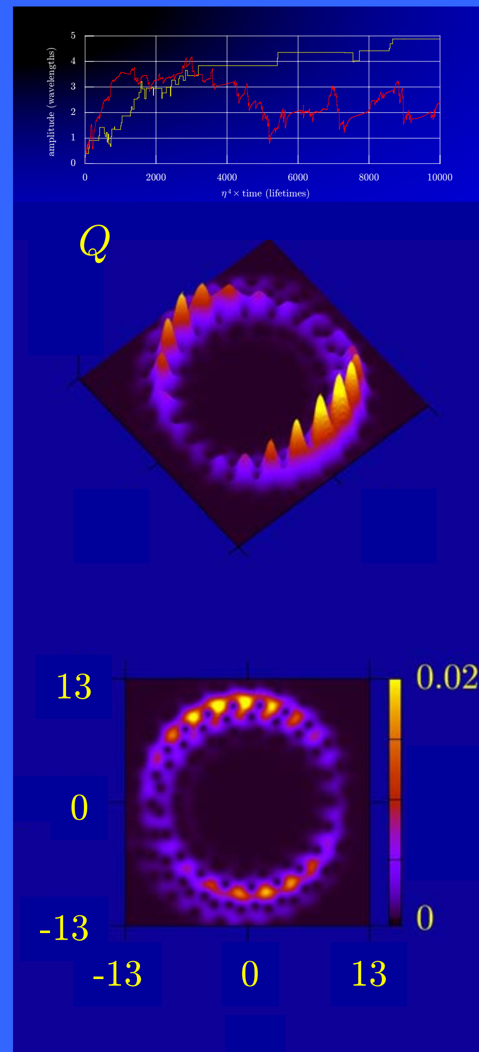
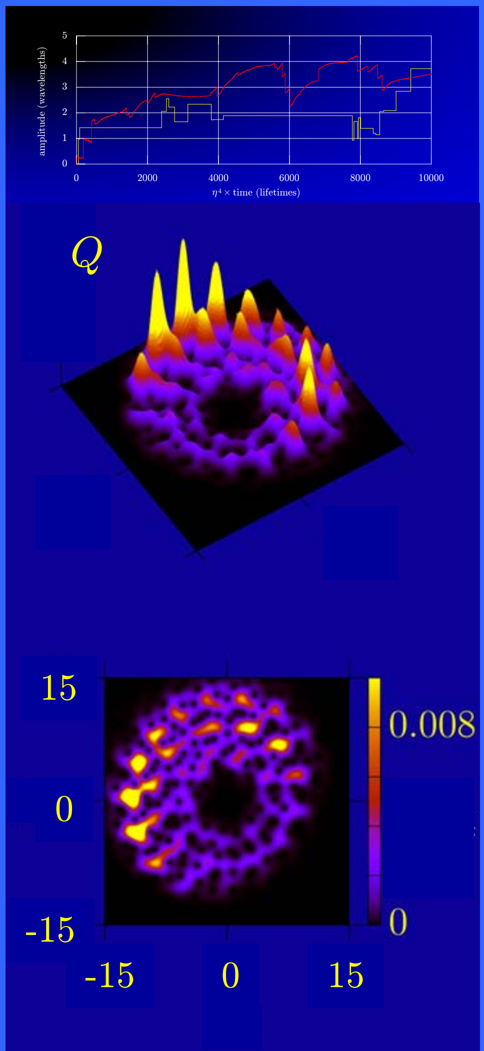
heating rates:



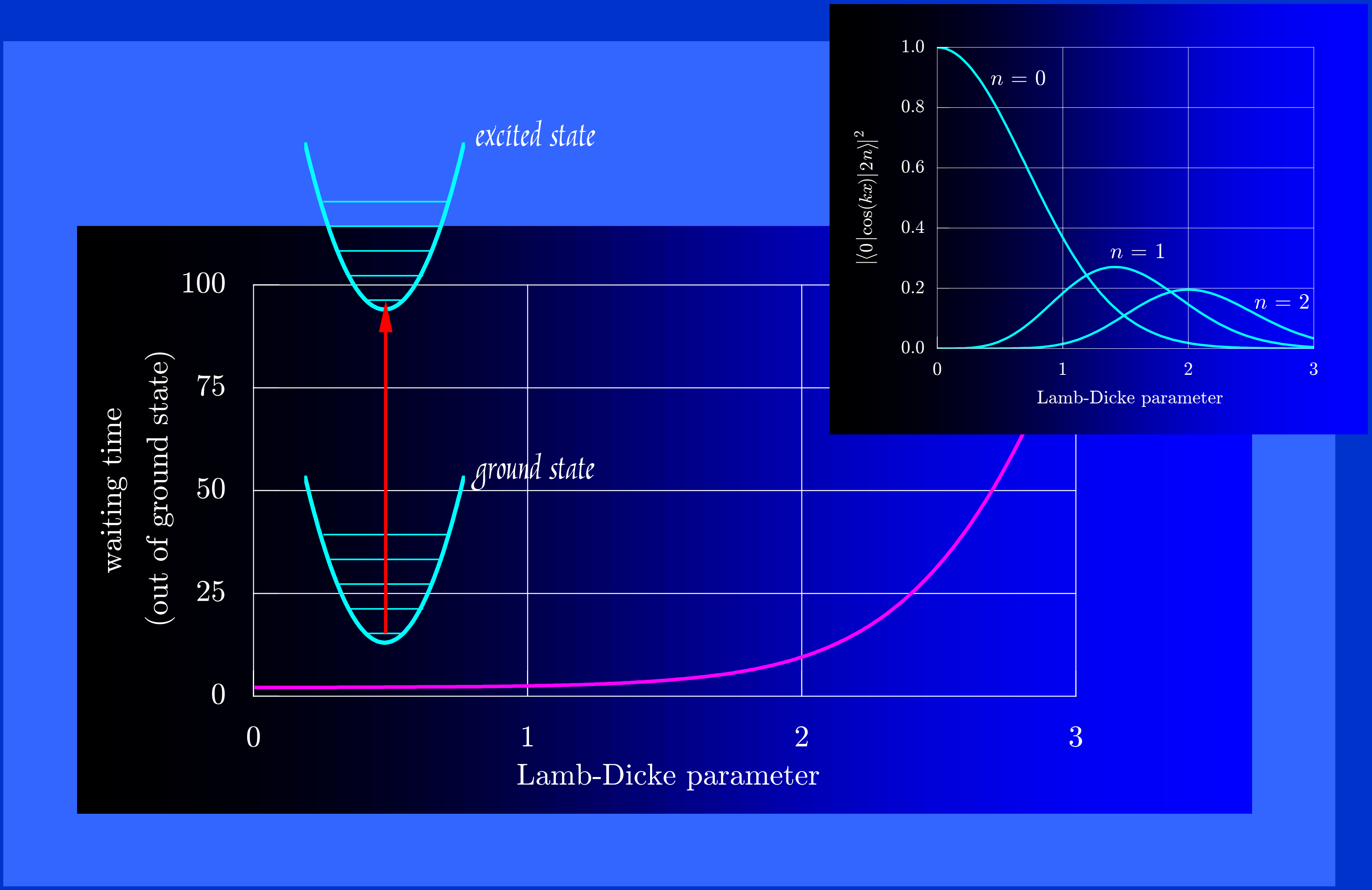
Lamb-Dicke parameter: 0.8 1.4 2.2







inhibition of fluorescence:



Quantum Stochastic Heating
Of a Trapped Ion

motion treated without semiclassical or other restrictions

exhibited nonperturbative dynamics (relying on the interplay of quantum coherence and decoherence)

treatment ranges from the quasi-classical (diffusion) limit to the manifestly quantum regime (large single-quantum events)